MATHEMATICAL MODELLING OF CONTINUOUS CASTING OF STEEL STRIPS BY SINGLE ROLL PROCESS

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PROCESS DESCRIPTION

Schematic representation of the Single Roll Continuous Casting of Steel Strips is shown in Figure 1. The liquid metal at a particular temperature T_1 is held in the reservoir where the liquid metal level is always kept constant. The metal continuously enters the annular space between the rotating chilled drum and the nozzle block through a graphite (refractory) nozzle. As soon as the molten metal comes in contact with the drum a skin of solid metal is formed which continues to grow as long as the liquid is in contact with this solidified skin. Water at an input temperature T_C extracts heat from the copper drum between the angles β_1 and β_2 . The drum rotates at an angular velocity W in anticlockwise direction. Skin formation begins at angle β_1 and completes at β_2 (see Figure 1). The liquid metal near the interface has a temperature T_p which is close to its freezing point. At the interface the following transformation reaction takes place:

$$\text{Liquid} \xrightarrow{\text{Latent heat}} \text{Solid}$$

MATHEMATICAL MODELLING

Macroscopic Model

A macroscopic model is described below which is based on the rate of growth of solidification front in the angular direction as a function of enthalpy. The Model predicts speed of rotation of the drum and the final strip thickness.

The model is based on the following assumptions :

i. The process is assumed to be at steady state.

ii. Heat flow occurs primarily in the angular and radial directions.

iii. Liquid metal at temperature T_1 is contained in the reservoir and it has constant height. Inside the pool, the liquid is perfectly mixed and hence at uniform temperature. This is designated by T_P which is the temperature of liquid adjacent to the liquid-solid interface.

iv. A particular overall heat transfer coefficient is assumed for the process and the equations solved for the above predictions. The heat transfer coefficient used here is between the liquid - solid interface to water.

v. The skin formation starts at angle β_1 and the final strip thickness t_f is obtained at angle β_2.

The model consists of the following equations:

(i) Angular relationship between the dimensional parameters.

(ii) Continuity equation for the description of process along with boundary conditions.

$$\frac{k}{\rho_S C_{pS}} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) \right] - W \frac{dT}{d\alpha} = 0$$

(For meaning of symbols please refer to the list of symbols at the end of this report.)

(iii) Development of solidified shell.

$$-k \frac{dT}{dr} - h_{MS}(T_P - T_F) = \rho_S \frac{dt}{d\alpha} W \left[H(T_P) - H_S(T_F) \right]$$

(Equation at the interface)

(iv) Heat balance on the liquid pool

(v) Solidification and removal of latent heat.

(vi) Analytical solution of the governing equation based on the assumption that the solidified shell thickness is small enough to make the thermal conductivity large.

The following final equation gives the strip thickness :

$$t^* = \left(\frac{R_d}{t_f} \right) (q - 1) \left(1 - \exp\left[- \left(\frac{t_f}{R_d} \right)\left(\frac{1}{q} \right) \left(t^* + \frac{h_{MS}(T^*_P - T^*_F)(\alpha^* - \beta^*_1)}{H^*(T^*_P) - H^*(T^*_F)} \right)\right]\right)$$

where $$q = \frac{H_{SW} (T^*_F - T^*_W)}{H^*_{MS}(T^*_P - T^*_F)}$$

Dimensionless thickness $t^* = t/t_f$

Dimensionless temperature $T_i^* = \dfrac{T_X - T_S}{T_L - T_S}$

where subscript i may be P, F, l or W refering to liquid metal pool adjacent to drum, freezing, liquid metal in the resevoir, and water, respectively.

Dimensionless heat transfer coefficient $h_j^* = \dfrac{2\pi h_j}{c_{pS}\ \rho_S\ W\ t_f}$

where the subscript j may be MS or SW.

Dimensionless Entholpy at temperature T^*, $H^*(T^*) = \dfrac{H(T)}{c_{pS}\ (T_L - T_S)}$

Dimensionless heat capacity, $c_p^* = c_p/c_{pS}$

Dimensionless latent heat, $L^* = \dfrac{L}{c_{pS}\ (T_L - T_S)}$

Dimensionless angle $\theta^* = \theta\ /2\pi$

where θmay refer to β_1, β_2 or α

All equations are expressed in the form of dimensionless variables. The equations are solved analytically for various sets of input conditions which include :

i. Superheat of liquid

ii. Approximated freezing temperature for the alloy

iii. R.P.M. of the roll

iv. Final strip thickness

v. Heat transfer coefficients

Since at the present moment the heat transfer coefficient between the water spray and copper drum surface is not known the model equations have been solved for a wide range of heat trnasfer coefficients. The other data used in the model are given below.

Heat capacity of liquid (steel) : 866.0 $W\ kg^{-1}\ {}^oC^{-1}$

Heat capacity of solid (steel) : 690.0 $W\ kg^{-1}\ {}^oC^{-1}$

Latent Heat evolved	: 272142.0	$W\ kg^{-1}$
Melt-Interface ht.tr.coefficient	: 10000.0	$W\ m^{-2}\ {}^{o}C^{-1}$
Steel thermal conductivity	: 16.0	$W\ m^{-1}\ {}^{o}C^{-1}$
Copper thermal conductivity	: 380.0	$W\ m^{-1}\ {}^{o}C^{-1}$

The solution of model equations results in selecting that heat transfer coefficient which would give the assumed final strip thickness at β_2 and confirms the R.P.M. which made it possible. A few typical results obtained using this model are presented below.

Figure 2 represents the operating conditions required to produce a strip of final thickness of 0.001 m. Figure 3 is a similar representation of a strip of final thickness of 0.005 m. A typical computer output showing these results in the tabular form is shown in Table I.

Similar results have also been obtained for the following operating conditions :

i. Strip thickness : 0.002, 0.003, 0.01 m

ii. Different superheats : 1650 & 1700 ^{o}C

Microscopic Model Based on Enthalpy Balance

The macroscopic model described above presents only a simplified description of the process. To present a more realistic situation a microscopic model based on overall enthalpy balance is developed.

To develop such a model the various process components are discretised into several small elements as shown in Figure 4. It can be seen that the solidified strip is divided into 100 cylindrical elements of equal width along its length and 10 equal elements at the exit point of the strip from the liquid pool. The Copper drum is similarly divided into 100 equal elements along the length and 50 elements along the width at exit point.

The model is based on the following assumptions :

i. An initial thickness t_i is obtained as soon as the liquid strikes the drum at β_1. The final thickness t_f is attained at β_2. Between β_1 and β_2 the thickness increases linearely.

ii. Heat transfer coefficient between copper drum and water is specified for the particular spray nozzle geometry and the water flow rate.

iii. Constant high temperature thermal conductivity value prevails in the solid.

iv. Freezing temperature of the liquid alloy is assumed to be constant.

v. There is perfect thermal contact between Copper drum and strip and there is no slip between the two during the rotation of the drum.

The model involves enthalpy balance along the boundaries of the system comprising of elements which results in 10 algebraic equations.

The solution of model equation results in predicting the required speed of rotation of the drum for a strip of specified thickness. It also predicts the complete temperature profile in :

(i) the strip, while it is growing

(ii) the Copper drum

As in previous case the computations are being carried out for various assume values of heat transfer coefficient between the water spray and the inne surface of Copper drum. The results are being predicted for :

(i) strip thicknesses of 0.003, 0.005, 0.010 m

(ii) Liquid metal temperature ranging between 1550 and 1650 ^oC

(iii) Heat transfer coefficients (Water-inner surface of Cu drum) varying betwee 1000 - 100000 W m^{-2} $^oC^{-1}$.

Microscopic Model Based on Mass, Momentum and Heat Balance

As a next step to the microscopic model based on enthalpy balance, we hav undertaken the development of a model based on microscopic balance of mass momentum and heat balance.

The model involves a set of partial differential equations of Second order These equations are valid for the three regions:

(1) solidifying strip, (2) solid/liquid interface and (3) liquid metal poo contained between the nozzle block and the drum.

An important aspect of the model is that the velocity field in the annular space between the nozzle block and drum is predicted by solving :

(i) Conservation of momentum equation

(ii) Conservation of mass equation

(iii) Conservation of energy equation
(Navier-Stokes Equation)

For the process under observation these equations reduce to a two dimensional form. Further it is assumed that there is no radial gradient for pressure - this reduces the momentum balance to 1 - dimension.

By solving the momentum balance equation, the velocity field in the liquid region can be mapped and the convective heat transfer quantified.

The boundary conditions to solve the model equation involve :

(i) An overall heat transfer coefficient from the solidified strip to the cooling water in the spray.

(ii) At the refractory-liquid interface heat flow outwards is taken as zero, because the refractory is assumed to be a perfect insulator.

(iii) At the solid/liquid interface the exact location of the solid boundary is determined by interpolation and the latent heat evolved at this reaction zone accounted for.

Solution of Model Equation

Finite difference scheme is employed to solve the partial differential equations. After substituting the finite differences for the partial derivatives, a set of linear Ist order simultaneous equations are obtained. These are solved using IMPROVED GAUSSIAN ALGORITHM. The solution gives the temperature at various points.

After knowing the temperature distribution, the phase boundaries are located by running a subprogram, and the solidified strip thickness development from the start to the final value calculated.

The model can predict the following :

i. Strip thickness as a function of speed of rotation of the drum.

ii. Strip thickness as a function of liquid superheat.

iii. Strip thickness as function of heat transfer coefficient.

iv. Limiting conditions for the process.

LIST OF SYMBOLS

Temperatures:

T Variable temperature, K

T_1 Temperature of liquid in reservoir, K

T_P Temperasture of liquid in pooladjacent to the solid/liquid interface, K

T_F Freezing temperature of liquid, K

T_L Liquidus temperature, K

T_S Solidus temperature, K

T_w Temperature of water, K

T_c Inner surface temperature of the drum, K

Geometrical Dimensions

t_i Initial Strip thickness at angle β_1, m

t_f Final strip thickness at angle β_2, m

t Variable strip thickness between β_1 and β_2, m

α Variable angle, radians

R_d Radius of drum, m

β_1 Angle at which solidification starts, radians

β_2 Angle at which strip attains final thickness, radians

r Variable radius, m; $R_d < r < R_d+t$

h Height of liquid in reservoir, m

x Entry radius for liquid at nozzle, m

y Exit radius for liquid at nozzle, m

W Angular velocity of drum, radian/sec

Thermal Quantities

c_{pS} Heat capacity of solid, J $kg^{-1}\,{}^oC^{-1}$

c_{pl} Heat capacity of liquid, J $kg^{-1}\,{}^oC^{-1}$

H[ ] Enthalpy at the temperature mentioned, J kg^{-1}

L Latent heat of liquid-solid transformation, J kg^{-1}

h_{SW} Heat transfer coefficient between liquid-solid interface to water, W m^{-2} $^{o}C^{-1}$

h_{CW} Heat transfer coefficient between inner surface of Cu drum and water, W m^{-2} $^{o}C^{-1}$

h_{MW} Heat transfer coefficient between liquid metal and interface, W m^{-2} $^{o}C^{-1}$

k Thermal conductivity of solid, W m^{-2} $^{o}C^{-1}$

Note: Starred quantities refer to the dimensionless forms defined in the text.

TABLE 1

TABLE 1A : A typical computer output showing the results of macroscopic heat balance model

STRIP THICKNESS = 0.001 METRES

LIQUID TEMP DEG.C	OVERALL HT. TR.COEF. W/SQ.M.C	R.P.M.	POOL TEMP. DEG.C	ANGLE β_2 RADIANS
1600.0	780.00	3.00	1533.9	2.190
1600.0	820.00	3.00	1533.9	2.147
1600.0	860.00	3.00	1533.9	2.108
1600.0	900.00	3.00	1533.9	2.074
1600.0	940.00	3.00	1533.9	2.043
1600.0	980.00	3.00	1533.9	2.016
1600.0	1020.00	3.00	1533.9	1.990
1600.0	1060.00	3.00	1533.9	1.966
1600.0	1100.00	3.00	1533.9	1.945
1600.0	1140.00	3.00	1533.9	1.925
1600.0	1180.00	3.00	1533.9	1.907
1600.0	1220.00	3.00	1533.9	1.891
1600.0	1260.00	3.00	1533.9	1.875
1600.0	1300.00	3.00	1533.9	1.861
1600.0	1340.00	3.00	1533.9	1.847
1600.0	1380.00	3.00	1533.9	1.834
1600.0	1420.00	3.00	1533.9	1.822
1600.0	1460.00	3.00	1533.9	1.811
1600.0	1500.00	3.00	1533.9	1.801
1600.0	1540.00	3.00	1533.9	1.790
1600.0	1020.00	4.00	1538.5	2.207
1600.0	1060.00	4.00	1538.5	2.173
1600.0	1100.00	4.00	1538.5	2.142
1600.0	1140.00	4.00	1538.5	2.114
1600.0	1180.00	4.00	1538.5	2.088
1600.0	1220.00	4.00	1538.5	2.064
1600.0	1260.00	4.00	1538.5	2.042
1600.0	1300.00	4.00	1538.5	2.021
1600.0	1340.00	4.00	1538.5	2.001
1600.0	1380.00	4.00	1538.5	1.983
1600.0	1420.00	4.00	1538.5	1.966
1600.0	1460.00	4.00	1538.5	1.950
1600.0	1500.00	4.00	1538.5	1.935
1600.0	1540.00	4.00	1538.5	1.921
1600.0	1300.00	5.00	1542.4	2.189
1600.0	1340.00	5.00	1542.4	2.164
1600.0	1380.00	5.00	1542.4	2.140
1600.0	1420.00	5.00	1542.4	2.117
1600.0	1460.00	5.00	1542.4	2.097
1600.0	1500.00	5.00	1542.4	2.076
1600.0	1540.00	5.00	1542.4	2.058
1600.0	1540.00	6.00	1545.9	2.200

TABLE 1 CONTINUED ...

TABLE 1B : A typical computer output showing the results of macroscopic heat balance model

STRIP THICKNESS = 0.001 METRES

LIQUID TEMP DEG.C	OVERALL HT. TR.COEF. W/SQ.M.C	R.P.M.	POOL TEMP. DEG.C	ANGLE β_2 RADIANS
1650.0	580.00	2.00	1536.6	2.197
1650.0	620.00	2.00	1536.6	2.134
1650.0	660.00	2.00	1536.6	2.081
1650.0	700.00	2.00	1536.6	2.035
1650.0	740.00	2.00	1536.6	1.995
1650.0	780.00	2.00	1536.6	1.961
1650.0	820.00	2.00	1536.6	1.931
1650.0	860.00	2.00	1536.6	1.904
1650.0	900.00	2.00	1536.6	1.880
1650.0	940.00	2.00	1536.6	1.858
1650.0	980.00	2.00	1536.6	1.838
1650.0	1020.00	2.00	1536.6	1.820
1650.0	1060.00	2.00	1536.6	1.804
1650.0	1100.00	2.00	1536.6	1.789
1650.0	1140.00	2.00	1536.6	1.775
1650.0	1180.00	2.00	1536.6	1.763
1650.0	1220.00	2.00	1536.6	1.751
1650.0	1260.00	2.00	1536.6	1.740
1650.0	1300.00	2.00	1536.6	1.730
1650.0	1340.00	2.00	1536.6	1.720
1650.0	1380.00	2.00	1536.6	1.712
1650.0	1420.00	2.00	1536.6	1.703
1650.0	1460.00	2.00	1536.6	1.696
1650.0	1500.00	2.00	1536.6	1.688
1650.0	1540.00	2.00	1536.6	1.682
1650.0	860.00	3.00	1545.0	2.209
1650.0	900.00	3.00	1545.0	2.165
1650.0	940.00	3.00	1545.0	2.126
1650.0	980.00	3.00	1545.0	2.091
1650.0	1020.00	3.00	1545.0	2.060
1650.0	1060.00	3.00	1545.0	2.031
1650.0	1100.00	3.00	1545.0	2.005
1650.0	1140.00	3.00	1545.0	1.982
1650.0	1180.00	3.00	1545.0	1.960
1650.0	1220.00	3.00	1545.0	1.940
1650.0	1260.00	3.00	1545.0	1.921
1650.0	1300.00	3.00	1545.0	1.904
1650.0	1340.00	3.00	1545.0	1.888
1650.0	1380.00	3.00	1545.0	1.873
1650.0	1420.00	3.00	1545.0	1.859
1650.0	1460.00	3.00	1545.0	1.846
1650.0	1500.00	3.00	1545.0	1.833
1650.0	1540.00	3.00	1545.0	1.822
1650.0	1140.00	4.00	1552.3	2.214

TABLE 1B CONTINUED ...

STRIP THICKNESS = 0.001 METRES

LIQUID TEMP DEG.C	OVERALL HT. TR.COEF. W/SQ.M.C	R.P.M.	POOL TEMP. DEG.C	ANGLE β_2 RADIANS
1650.0	1180.00	4.00	1552.3	2.181
1650.0	1220.00	4.00	1552.3	2,151
1650.0	1260.00	4.00	1552.3	2.123
1650.0	1300.00	4.00	1552.3	2.097
1650.0	1340.00	4.00	1552.3	2.073
1650.0	1380.00	4.00	1552.3	2.051
1650.0	1420.00	4.00	1552.3	2.031
1650.0	1460.00	4.00	1552.3	2.011
1650.0	1500.00	4.00	1552.3	1.993
1650.0	1540.00	4.00	1552.3	1.976
1650.0	1460.00	5.00	1558.6	2.191
1650.0	1500.00	5.00	1558.6	2.166
1650.0	1540.00	5.00	1558.6	2.143

TABLE 1 CONTINUED ...

TABLE 1C : A typical computer output showing the results of macroscopic heat balance model

STRIP THICKNESS = 0.001 METRES

LIQUID TEMP DEG.C	OVERALL HT. TR.COEF. W/SQ.M.C	R.P.M.	POOL TEMP. DEG.C	ANGLE β_2 RADIANS
1700.0	340.00	1.00	1531.1	2.140
1700.0	380.00	1.00	1531.1	2.038
1700.0	420.00	1.00	1531.1	1.962
1700.0	460.00	1.00	1531.1	1.904
1700.0	500.00	1.00	1531.1	1.857
1700.0	540.00	1.00	1531.1	1.819
1700.0	580.00	1.00	1531.1	1.788
1700.0	620.00	1.00	1531.1	1.761
1700.0	660.00	1.00	1531.1	1.738
1700.0	700.00	1.00	1531.1	1.719
1700.0	740.00	1.00	1531.1	1.702
1700.0	780.00	1.00	1531.1	1.686
1700.0	820.00	1.00	1531.1	1.673
1700.0	860.00	1.00	1531.1	1.661
1700.0	900.00	1.00	1531.1	1.650
1700.0	940.00	1.00	1531.1	1.641
1700.0	980.00	1.00	1531.1	1.632
1700.0	1020.00	1.00	1531.1	1.624
1700.0	1060.00	1.00	1531.1	1.616
1700.0	1100.00	1.00	1531.1	1.610
1700.0	1140.00	1.00	1531.1	1.603
1700.0	1180.00	1.00	1531.1	1.597
1700.0	1220.00	1.00	1531.1	1.592
1700.0	1260.00	1.00	1531.1	1.587
1700.0	1300.00	1.00	1531.1	1.582
1700.0	1340.00	1.00	1531.1	1.578
1700.0	1380.00	1.00	1531.1	1.574
1700.0	1420.00	1.00	1531.1	1.570
1700.0	1460.00	1.00	1531.1	1.567
1700.0	1500.00	1.00	1531.1	1.563
1700.0	1540.00	1.00	1531.1	1.560
1700.0	660.00	2.00	1544.6	2.172
1700.0	700.00	2.00	1544.6	2.114
1700.0	740.00	2.00	1544.6	2.065
1700.0	780.00	2.00	1544.6	2.022
1700.0	820.00	2.00	1544.6	1.985
1700.0	860.00	2.00	1544.6	1.953
1700.0	900.00	2.00	1544.6	1.924
1700.0	940.00	2.00	1544.6	1.898
1700.0	980.00	2.00	1544.6	1.875
1700.0	1020.00	2.00	1544.6	1.854
1700.0	1060.00	2.00	1544.6	1.835

TABLE 1C CONTINUED ...

STRIP THICKNESS = 0.001 METRES

LIQUID TEMP DEG.C	OVERALL HT. TR.COEF. W/SQ.M.C	R.P.M.	POOL TEMP. DEG.C	ANGLE β_2 RADIANS
1700.0	1100.00	2.00	1544.6	1.818
1700.0	1140.00	2.00	1544.6	1.802
1700.0	1180.00	2.00	1544.6	1.788
1700.0	1220.00	2.00	1544.6	1.775
1700.0	1260.00	2.00	1544.6	1.762
1700.0	1300.00	2.00	1544.6	1.751
1700.0	1340.00	2.00	1544.6	1.740
1700.0	1380.00	2.00	1544.6	1.730
1700.0	1420.00	2.00	1544.6	1.721
1700.0	1460.00	2.00	1544.6	1.712
1700.0	1500.00	2.00	1544.6	1.704
1700.0	1540.00	2.00	1544.6	1.697
1700.0	980.00	3.00	1556.1	2.183
1700.0	1020.00	3.00	1556.1	2.144
1700.0	1060.00	3.00	1556.1	2.108
1700.0	1100.00	3.00	1556.1	2.076
1700.0	1140.00	3.00	1556.1	2.047
1700.0	1180.00	3.00	1556.1	2.020
1700.0	1220.00	3.00	1556.1	1.996
1700.0	1260.00	3.00	1556.1	1.974
1700.0	1300.00	3.00	1556.1	1.953
1700.0	1340.00	3.00	1556.1	1.934
1700.0	1380.00	3.00	1556.1	1.917
1700.0	1420.00	3.00	1556.1	1.900
1700.0	1460.00	3.00	1556.1	1.885
1700.0	1500.00	3.00	1556.1	1.871
1700.0	1540.00	3.00	1555.1	1.857
1700.0	1300.00	4.00	1566.1	2.189
1700.0	1340.00	4.00	1566.1	2.159
1700.0	1380.00	4.00	1566.1	2.132
1700.0	1420.00	4.00	1566.1	2.106
1700.0	1460.00	4.00	1566.1	2.083
1700.0	1500.00	4.00	1566.1	2.061
1700.0	1540.00	4.00	1566.1	2.040

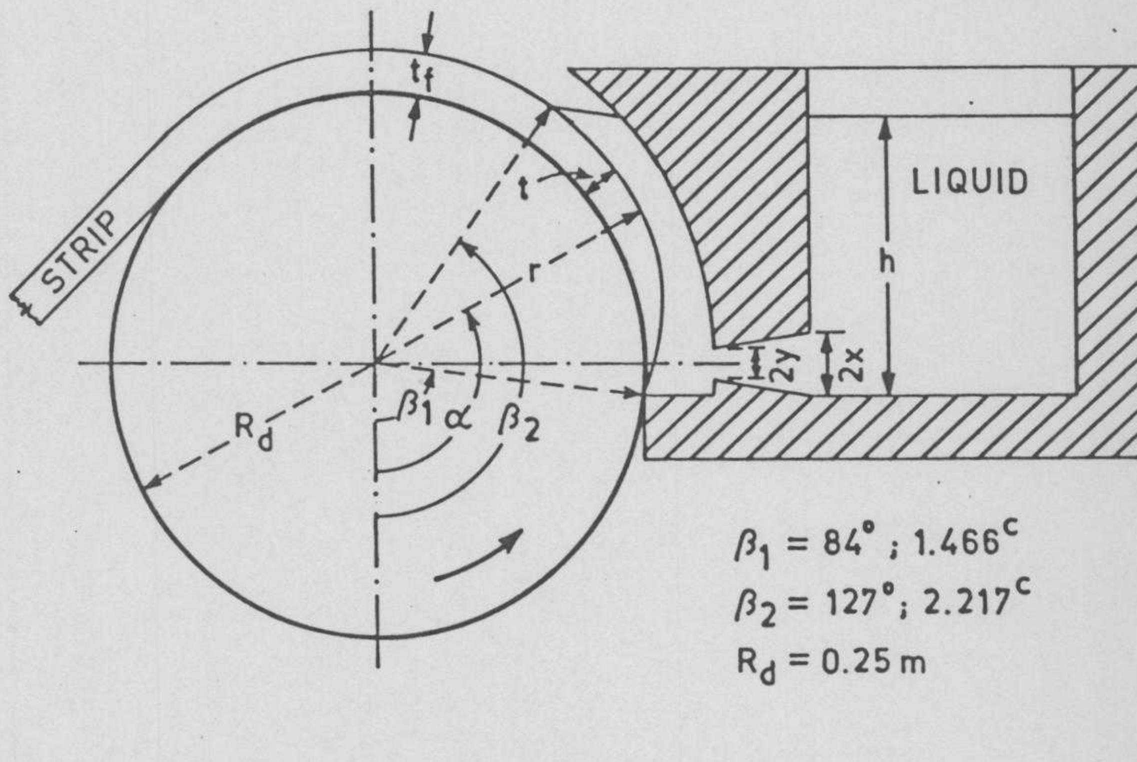

FIGURE 1 : Schematic representation of one roll strip caster

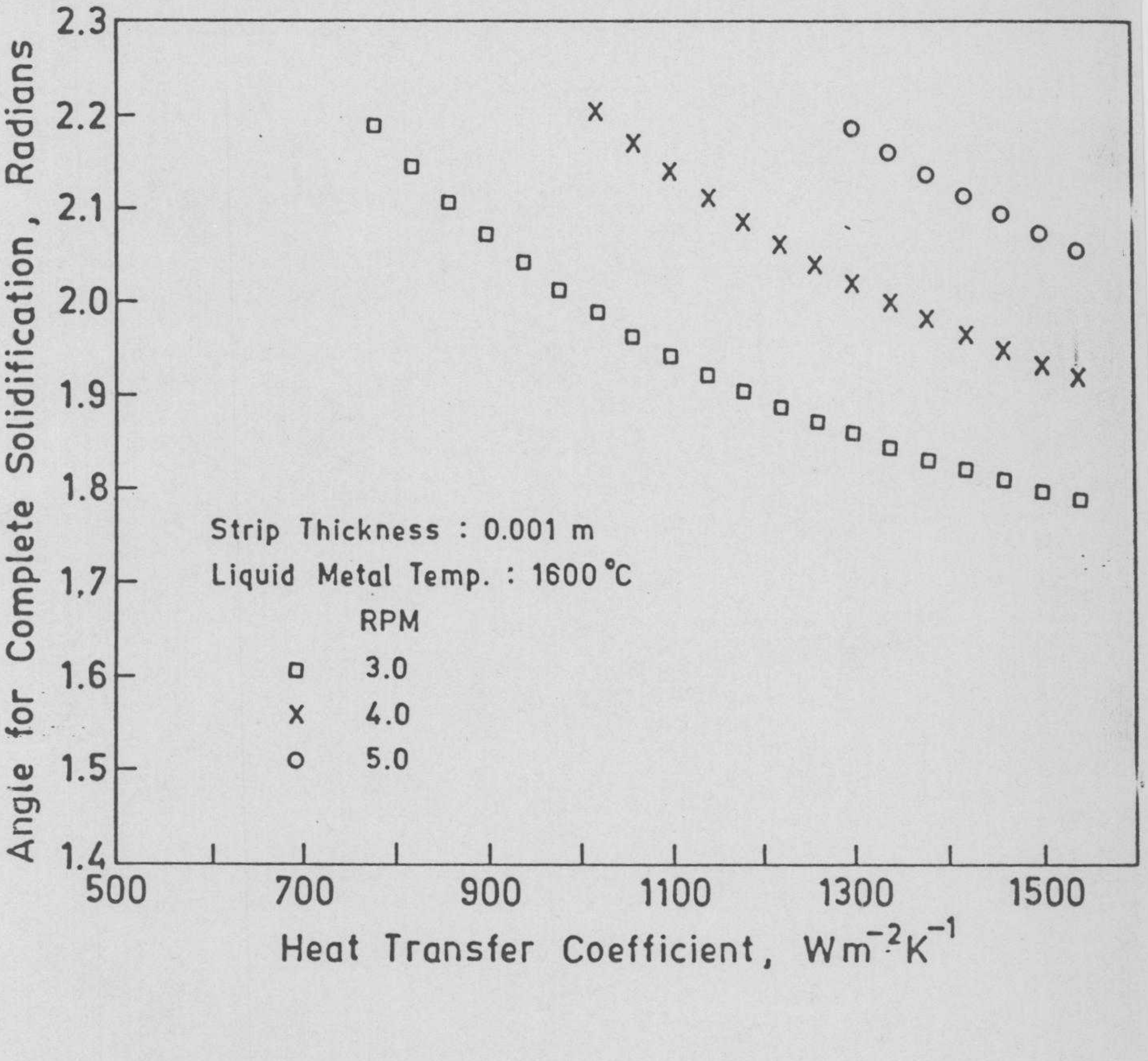

FIGURE 2 : Operating conditions required to produce a strip of thickness 0.001 metres

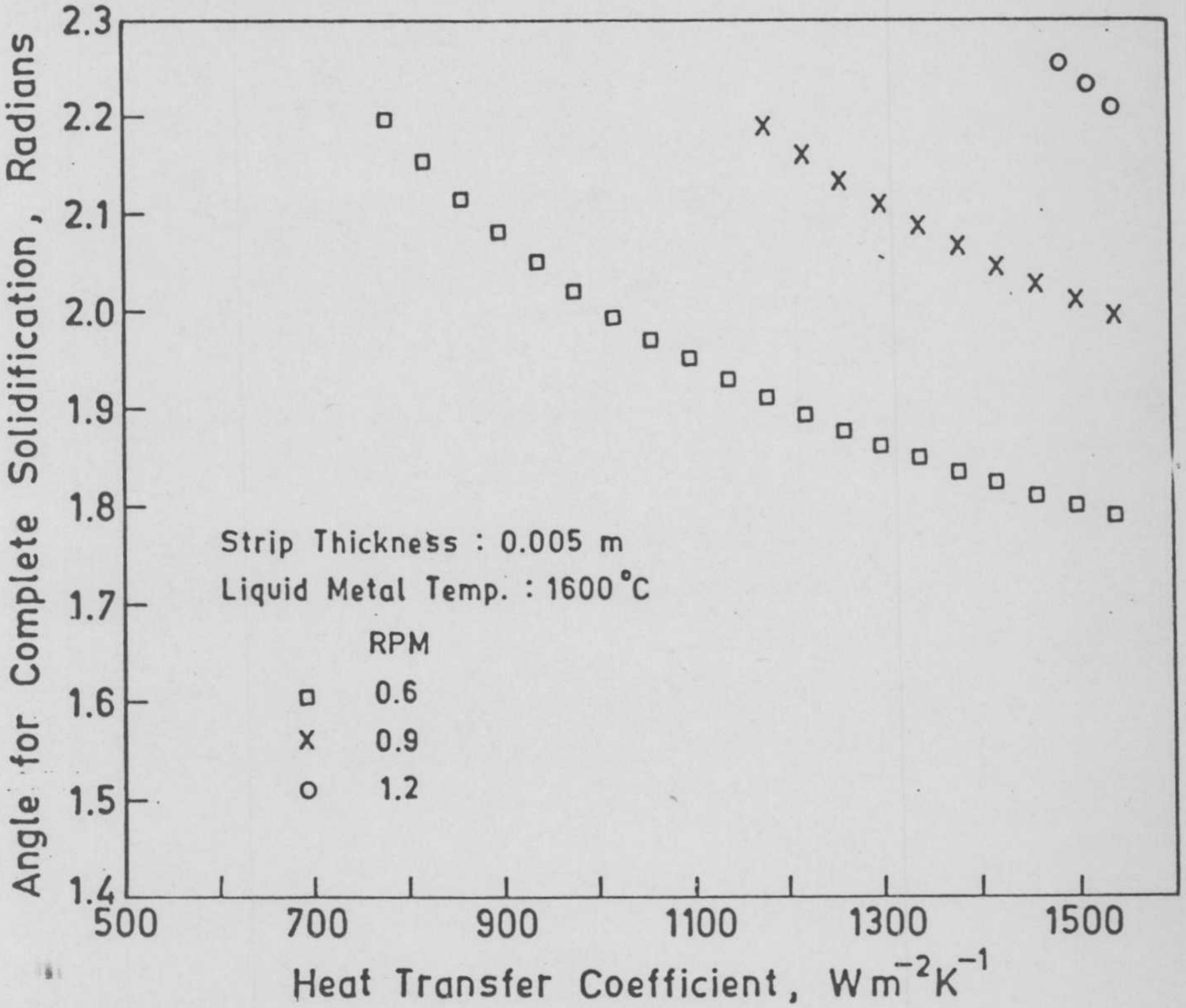

FIGURE 3 : Operating conditions required to produce a strip of thickness 0.005 metres

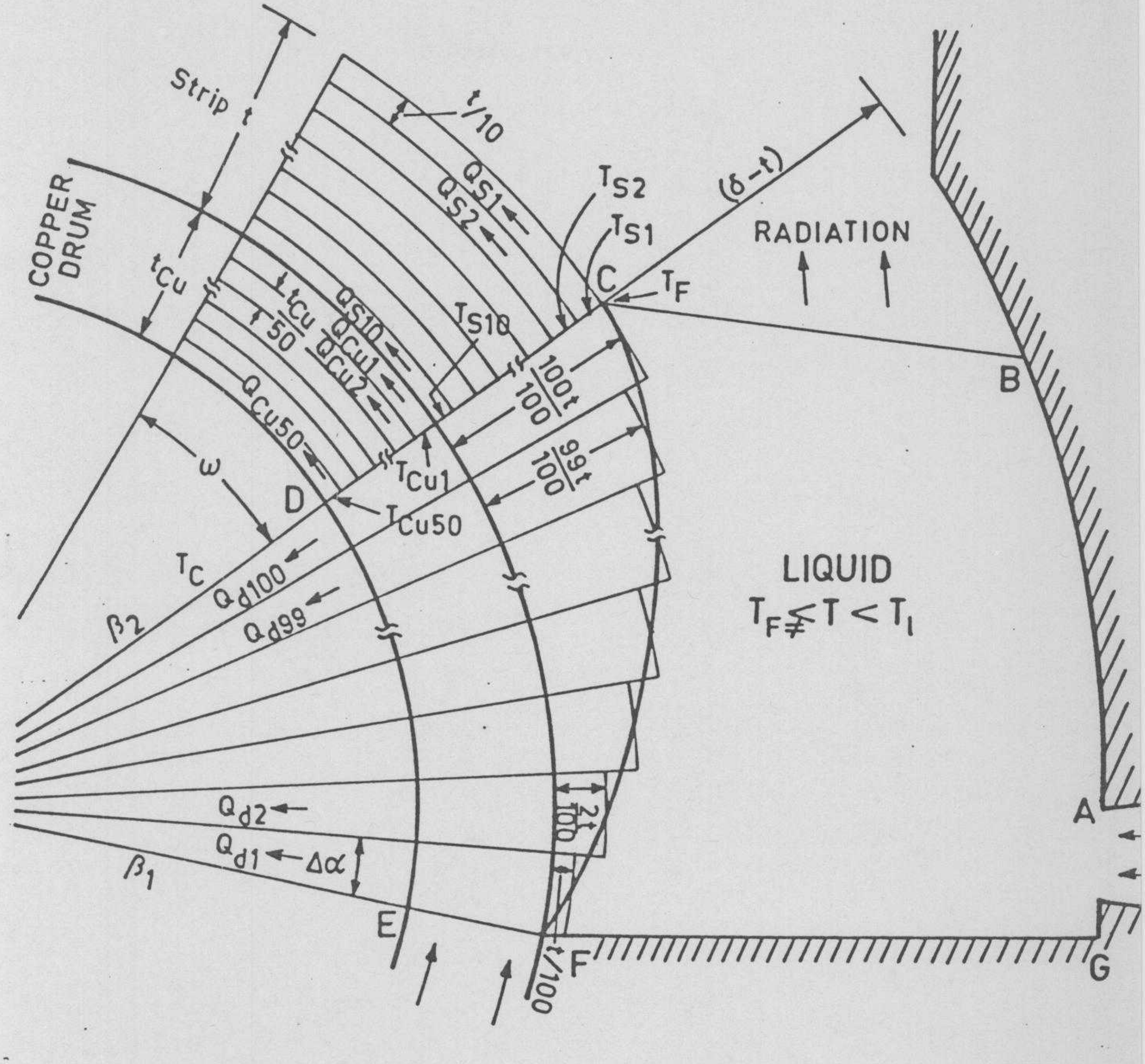

FIGURE 4 : Schematic representation of microscopic model based on enthalpy balance