Neuro-Optimal Control of Helicopter UAVs

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ABSTRACT

Helicopter UAVs can be extensively used for military missions as well as in civil operations, ranging from multi-role combat support and search and rescue, to border surveillance and forest fire monitoring. Helicopter UAVs are underactuated nonlinear mechanical systems with correspondingly challenging controller designs. This paper presents an optimal controller design for the regulation and vertical tracking of an underactuated helicopter using an adaptive critic neural network framework. The online approximator-based controller learns the infinite-horizon continuous-time Hamilton-Jacobi-Bellman (HJB) equation and then calculates the corresponding optimal control input that minimizes the HJB equation forward-in-time. In the proposed technique, optimal regulation and vertical tracking is accomplished by a single neural network (NN) with a second NN necessary for the virtual controller. Both of the NNs are tuned online using novel weight update laws. Simulation results are included to demonstrate the effectiveness of the proposed control design in hovering applications.

Keywords: Nonlinear optimal control, helicopter UAV, neural network (NN), online approximator (OLA), HJB equation, hovering

1. INTRODUCTION

Helicopter UAVs have many capabilities such as vertical take-off, hovering, low-speed flight at low altitude, and landing. For control of a helicopter,[1] it is necessary to produce moments and forces on the vehicle with two goals: first, to position the helicopter in equilibrium such that the desired trim state is achieved, and second, to control the helicopter's velocity, position and orientation such that it hovers as desired with minimum error. The dynamics of the helicopter UAV are not only nonlinear but also coupled with each other and underactuated, which makes the UAV difficult to control. The helicopter has six degrees of freedom (DOF) which must be controlled with only four control inputs - thrust and the three rotational torques.

To solve this control problem, several techniques have been proposed[1–7] for model-based control of helicopter UAVs. It has been shown[1] that the multivariable nonlinear helicopter model cannot be converted into a controllable linear system via exact state space linearization. In addition, for certain output functions, exact input-output linearization results in unstable zero dynamics.[8] Based on Newton-Euler equations, a dynamic model has been derived[1] considering the helicopter as a rigid body with input forces and torques applied to the center of mass. Previous researchers have considered an adaptive output feedback control of uncertain nonlinear system with unknown dynamics and dimensions, a controller for autonomous helicopter flight, with the control problem[3] separated into an inner loop attitude control and outer loop trajectory control. A drawback of these controllers[2–4] is that the coupling between rolling (pitching) moments and lateral (longitudinal) accelerations are completely neglected. A backstepping-based controller has been presented[5] for the autonomous landing of the rotary wing helicopter which also holds good for full flight control. The nonlinear controller computes the

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desired thrusts and flapping angles to get the commanded position and then computes the control inputs which achieve the desired thrust and flapping angles. Models which are properly trained offline are often robust to small variations in the system but fail to adapt to larger changes in the system. Further, an offline scheme alone does not allow the NN to learn any new dynamics it encounters during a new maneuver. NN approaches[2] have been proposed to learn the dynamics of the unmanned helicopter online, but the observer used in this case estimates only the states of the feedback linearized system and not the actual states of the helicopter dynamics. A nonlinear controller for a quadrotor unmanned aerial vehicle has been proposed[7] by employing output feedback and NNs. It has been assumed that the availability of the dynamics of the UAV is not always feasible and therefore a NN has been introduced to learn the complete dynamics of the unmanned quadrotor online, also including the uncertain nonlinear terms such as aerodynamic drag. A single online approximator (SOLA)-based scheme has been introduced[9] to solve the optimal regulation and tracking control problems for affine nonlinear continuous-time systems with known dynamics. The SOLA-based adaptive approach has been designed to learn the infinite horizon continuous-time HJB equation, and the corresponding optimal control input that minimizes the HJB equation has been calculated forward-in-time.

However, the optimal controller design for regulation and vertical tracking of an underactuated helicopter using NN has not yet been attempted, to the best of the authors' knowledge. Following a previous approach,[9] in this paper the SOLA-based scheme for optimal regulation and vertical tracking of a nonlinear continuous-time strict feedback system with known dynamics has been considered. The online approximator-based dynamic controller learns the continuous-time Hamilton-Jacobi-Bellman (HJB) equation and then calculates the corresponding optimal control input that would minimize the HJB equation forward-in-time. This SOLA-based optimal control scheme is then extended for the optimal regulation and vertical tracking of a helicopter UAV with known dynamics. The proposed controller will consist of three NNs - one for approximating the cost function, with a second NN necessary for the virtual controller, a third NN used for tracking during take-off and landing, and all the NNs tuned online using novel weight update laws. The paper is organized as follows: the next section presents the nonlinear model of the helicopter, Section 3 deals with the continuous-time nonlinear optimal HJB regulation and tracking problem and the solution of the HJB equation forward-in-time, and the following sections include simulation results and concluding remarks.

2. DYNAMIC MODEL OF THE HELICOPTER

Consider a helicopter with six degrees of freedom (DOF) defined in the inertial coordinate frame $\mathcal{Q}^a$, where its position coordinates are given by $\rho = [x, y, z]^T \in \mathcal{Q}^a$ and its rotational orientation described as roll, pitch and yaw respectively, are given by $\Theta = [\phi, \theta, \psi]^T \in \mathcal{Q}^a$. The equations of motion can be expressed in the body fixed frame $\mathcal{Q}^b$ which has as its origin the center of mass of the helicopter. The ${}^b x$-axis is defined parallel to the helicopter's direction of travel, the ${}^b y$-axis is defined perpendicular to the helicopter's direction of travel, while the ${}^b z$-axis is defined as projecting orthogonally downwards from the xy-plane of the helicopter. The dynamics of the helicopter are given by the Newton-Euler equation in the body fixed frame and can be written as[1]

$$\begin{bmatrix} mI & 0 \\ 0 & \mathcal{J} \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \omega \times mv \\ \omega \times \mathcal{J}\omega \end{bmatrix} = \begin{bmatrix} F \\ \tau \end{bmatrix}$$

where

$m \in \mathbf{R}$ is the positive scalar denoting the mass of the helicopter,
$F \in \mathbf{R}^{3\times 1}$ is the body force applied to the helicopter's center of mass,
$\tau \in \mathbf{R}^{3\times 1}$ is the body torque applied to the helicopter's center of mass,
$v = [v_x, v_y, v_z]^T \in \mathbf{R}^{3\times 1}$ represents the translational velocity vector,
$\omega = [\omega_x, \omega_y, \omega_z]^T \in \mathbf{R}^{3\times 1}$ represents the body angular velocity vector,
$I \in \mathbf{R}^{3\times 3}$ is the identity matrix,

and $\mathcal{J} \in \mathbf{R}^{3\times 3}$ is the positive-definite inertia matrix.

The kinematics of the helicopter are given as in equations (1) and (2) in Dierks,[7] along with the translational rotation matrix T used to relate a vector in body fixed frame to the inertial coordinate frame, and the rotational transformation matrix R used to relate a vector in body fixed frame to the inertial coordinate frame. The transformation matrix is bounded according to $\|T\|_F < T_{max}$ for a known constant T_{max} provided $-\pi/2 < \phi < \pi/2$ and $-\pi/2 < \theta < \pi/2$ such that the helicopter trajectory does not pass through any singularities.[1]

Here it is necessary to mention that $\|R\|_F = R_{max}$ for a known constant R_{max} and $R^{-1} = R^T$. Let the mass-inertia matrix M and skew-symmetric matrix $\bar{S}(\omega)$ be given by Dierks.[7] Now the dynamics can be rewritten in the following form[7]

$$M \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} = \bar{S}(\omega) \begin{bmatrix} v \\ \omega \end{bmatrix} + \begin{bmatrix} \mathcal{N}_1(v) \\ \mathcal{N}_2(\omega) \end{bmatrix} + \begin{pmatrix} G(R) \\ 0^{3\times 1} \end{pmatrix} + U + \tau_d \tag{1}$$

where $\mathcal{N}_1(v)$, $\mathcal{N}_2(\omega) \in \mathbf{R}^{3\times 1}$ represents the nonlinear aerodynamic effects such as fuselage drag, $G(R) \in \mathbf{R}^{3\times 1}$ represents the gravity vector and is defined as $G(R) = [-mg\sin(\theta) \;\; mg\sin(\phi)\cos(\theta)$ $mg\cos(\phi)\cos(\theta)]^T$ with $g = 9.81 m/s^2$, $U = [u_1 \;\; u_2 \;\; u_3 \;\; u_4 \;\; u_5 \;\; u_6]^T \in \mathbf{R}^{6\times 1}$ is the control input vector, with u_1 providing the thrust in the z-direction, u_2, u_3 and u_4 providing the rotational torques in $x-$, $y-$ and $z-$ directions respectively, and $\tau_d = [\tau_{d1}^T, \tau_{d2}^T]^T$ represents unknown bounded disturbances such that $\|\tau_d\| < \tau_M$ for all time t, and τ_M being a known positive constant. The nonlinear aerodynamic effects taken into consideration for modeling of the helicopter are composed of translational aerodynamic drag and the main rotor torque that is offset by the tail rotor. Defining $X = [\rho^T \;\; \Theta^T]^T$ and $V = [v^T \;\; \omega^T]^T \in \mathbf{R}^{6\times 1}$, one can rewrite (4) employing backstepping in the strict-feedback form as

$$\dot{X} = A(t)V + \xi \tag{2}$$
$$\dot{V} = f(V) + \bar{U} \tag{3}$$

where $f(V) = M^{-1}(\bar{S}(\omega)V + [\mathcal{N}_1(v)\mathcal{N}_2(\omega)]^T) + \bar{G}$ with $\bar{G} = M^{-1}G(R) \in \mathbf{R}^{6\times 1}$, $\bar{U} = M^{-1}U$, $\xi = \xi_1 + \bar{\tau}_d$ with $\xi_1 \in \mathbf{R}^{6\times 1}$ being the bounded sensor measurement noise such that $\|\xi_1\| \leq \xi_{1M}$ for a known constant ξ_{1M} and $\bar{\tau}_d = [\bar{\tau}_{d1}^T, \bar{\tau}_{d2}^T]^T = [\tau_{d1}^T/m, (J^{-1}\tau_{d2})^T]^T \in \mathbf{R}^{6\times 1}$, and $A(t)$ as defined in Dierks.[7] In this section, the dynamic model of the helicopter with six degrees-of-freedom (DOF) and six inputs has been presented. The inputs are functions of main rotor thrust T_{MR}, tail rotor thrust T_{TR}, the longitudinal tilt α, and the lateral tilt β of the main rotor path plane with respect to the shaft.

In the case of this system, there is a high degree of coupling and nonlinearity in the inputs to the system dynamics, such that it is mathematically impossible for this model to uniquely determine the system dynamics inputs from the optimal controller-generated inputs. In order to control the system, it is necessary to reduce the system such that only four states are monitored and controlled. The trade-off is that a powerful nonlinear neural-network-based controller is used to evaluate a cost function online for optimality, but this can only be done by imposing a requirement that the helicopter remain in hover.

Now, consider the input U to be given by $U = [u_1 \;\; u_2 \;\; u_3 \;\; u_4]^T \in \mathbf{R}^{4\times 1}$ as the control input vector, with u_1 providing the thrust in the z-direction, and u_2, u_3 and u_4 providing the rotational torques in $x-$, $y-$ and $z-$ directions respectively. Consequently, in order to control the four inputs of the helicopter, from the next section onwards, only four states (z-axis translational velocity, and roll, pitch and yaw angular velocities) of the helicopter will be considered.

3. NONLINEAR OPTIMAL REGULATION & VERTICAL TRACKING OF THE HELICOPTER UAV

3.1 Hamilton-Jacobi-Bellman Equation

In this section, the dynamics of the helicopter given in (2) and (3) which are of the form

$$\dot{V} = f(V) + gu_V \tag{4}$$

will be considered, where $V \in \mathbf{R}^{4\times 1}$, $f(V) \in \mathbf{R}^{4\times 1}$, $g = M^{-1} \in \mathbf{R}^{4\times 4}$ is bounded such that $g_{min} \leq \|g\|_F \leq g_{max}$ and $u_V \in \mathbf{R}^{4\times 1}$ is the control input. It has been assumed that the system is observable and controllable, with $V = 0$ a unique equilibrium point on compact set $\Upsilon \in \mathbf{R}^{4\times 1}$ with $f(0) = 0$.[9] With these assumptions, the optimal control input for the unmanned helicopter system given in (4) can be determined.[11] It is important to note that the dynamics $f(V)$ and g are assumed to be known. However, this assumption may be relaxed if some of the unknown parameters are estimated by using NNs.

The infinite horizon HJB cost function for (4) is given below

$$W(V(t)) = \int_t^{\infty} r(V(\tau), u_V(\tau))d\tau \tag{5}$$

with $r(V(t), u_V(t)) = Q(V) + u_V^T B u_V$, $Q(V) > 0$ the positive definite penalty on the states and $B \in \mathbf{R}^{4\times 4}$ denoting a positive definite matrix. The control input must be selected such that the cost function in (5) is finite, or u_V must be admissible.[9] Next, the Hamiltonian for the cost function in (5) with control input u_V is defined as

$$H(V, u) = r(V, u) + W_V^T(V)(f(V) + gu_V) \tag{6}$$

with $W_V(V)$ the gradient of $W(V)$ with respect to V. Since the optimal control input u_V^* which minimizes the cost function in (5) will also minimize the Hamiltonian in (6). Thus the optimal control input can be obtained by solving the stationary condition $\partial H(V, u_V)/\partial u_V = 0$ and is found to be

$$u_V^*(V) = -B^{-1}g^T W_V^*(V)/2 \tag{7}$$

Substituting the optimal control input from (7) into the Hamiltonian (6) while retaining $H(V, u_V^*, W_V^*(V)) = 0$ gives the HJB equation and the necessary and sufficient condition for optimal control to be[11]

$$0 = Q(V) + W_V^{*T}(V)f(V) - W_V^{*T}(V)gB^{-1}g^T W_V^*(V)/4 \tag{8}$$

with $W^*(0) = 0$. It is also known that the following relation is applicable[9]

$$J_{1V}^T(f(V) + gu_V^*) = -J_{1V}^T \bar{Q}(V) J_{1V} \tag{9}$$

Tracking results are also possible by modifying [7] to obtain an expression for the optimal control input as given below

$$\hat{u}_V = u_d - B_e^{-1} g^T W_{Te}^*(e)/2$$

with the desired control input u_d expressed as

$$u_d(V_d) = g^{-1}(\dot{V}_d - f(V_d))$$

The control input consists of a predetermined feedforward term, u_d, and an optimal feedback term that is a function of the gradient of the optimal cost function. The details of this result will not be presented here, but may be found in [9]. It is important to note that successful implementation for trajectory tracking along the z-axis requires that the states V be replaced with the error between the actual and desired states e.

3.2 Single Online Approximator(SOLA)-Based Optimal Control of Helicopter

Usually, in adaptive critic based techniques, two OLAs[9] are used for optimal control, since one is used to approximate the cost function and the other is used to generate the control action. In this paper, the adaptive critic for optimal control of a helicopter is accomplished using only one OLA. For the SOLA to learn the cost function, the cost function is rewritten using the OLA representation as given below

$$W(V) = \Gamma^T \Phi(V) + \varepsilon(V) \tag{10}$$

with $\Gamma \in \mathbf{R}^{\mathbf{L}}$ the constant target OLA vector, $\Phi(V) : \mathbf{R}^n \to \mathbf{R}^L$ a linearly independent basis vector such that $\Phi(V) = 0$, with $\varepsilon(V)$ the OLA reconstruction error. The target OLA vector and reconstruction errors are assumed to be upper bounded, with $\|\Gamma\| \le \Gamma_M$ and $\|\varepsilon(V)\| \le \varepsilon_M$.[10] The OLA cost function gradient in (10) is

$$\partial W(V)/\partial V = W_V(V) = \nabla_V^T \Phi(V)\Gamma + \nabla_V \varepsilon(V) \tag{11}$$

Using (11), the optimal control input in (7) and the HJB equation in (8) can be expressed as

$$
\begin{aligned}
u_V^* &= -B^{-1}g^T\nabla_V^T\Phi(V)\Gamma/2 - B^{-1}g^T\nabla_V\varepsilon(V)/2 &(12)\\
H^*(V,\Gamma) &= Q(V)+\Gamma^T\nabla_V\Phi(V)f(V)-\Gamma^T\nabla_V\Phi(V)C\nabla_V^T\Phi(V)\Gamma/4+\varepsilon_{HJB}=0 &(13)
\end{aligned}
$$

where $C=gB^{-1}g^T>0$ is bounded with $C_{min}\le\|C(V)\|\le C_{max}$ for C_{min} and C_{max} and $\varepsilon_{HJB}=\nabla_V\varepsilon^T(f(V)-\frac{1}{2}gB^{-1}g^T(\nabla_V^T\Phi(V)\Gamma+\nabla_V\varepsilon))+\frac{1}{4}\nabla_V\varepsilon^TgB^{-1}g^T\nabla_V\,\varepsilon=\nabla_V\varepsilon^T(f(V)+gu_V^*)+\frac{1}{4}\nabla_V\varepsilon^TC\nabla_V\,\varepsilon$ is the OLA residual reconstruction error. The OLA estimate of (10) is

$$\hat{W}(V)=\hat{\Gamma}^T\Phi(V) \tag{14}$$

with $\hat{\Gamma}$ the OLA estimate of the target vector Γ. In the same way the estimate for the optimal control input in (12) can be expressed as $\hat{u}_V^*=-B^{-1}g^T\nabla_V^T\Phi(V)\hat{\Gamma}/2$. An initial stabilizing control is not required to implement this proposed SOLA-based scheme. Lyapunov analysis shows that the estimated control input approaches the optimal input with a bounded error.

$$\hat{H}^*(V,\hat{\Gamma})=Q(V)+\hat{\Gamma}^T\nabla_V\Phi(V)f(V)-\hat{\Gamma}^T\nabla_V\Phi(V)C\nabla_V^T\Phi(V)\hat{\Gamma}/4 \tag{15}$$

Remark 1: From the definition of the OLA cost function approximation (14) and the Hamiltonian function (15), it is clear that both become zero when $\|V\|=0$. Therefore, once the system states have converged to zero, the cost function approximation can no longer be updated, which satisfies the persistency of excitation (PE) requirement for the inputs to the cost function OLA.[10] Stated differently, the system states must be persistently exciting long enough for the OLA to learn the optimal cost function. Recollecting the HJB equation in (6), the OLA estimate $\hat{\Gamma}$ should be tuned such that $\hat{H}^*(V,\hat{\Gamma})$. Unfortunately, only tuning $\hat{\Gamma}$ to minimize $\hat{H}^*(V,\hat{\Gamma})$ does not guarantee the stability of the nonlinear helicopter system(4) throughout the OLA learning process.

Consequently, the OLA tuning algorithm is designed to minimize (15) while maintaining the stability of (4) and given as

$$
\begin{aligned}
\dot{\hat{\Gamma}} &= -\delta_1\frac{\hat{\beta}}{(\hat{\beta}^T\hat{\beta}+1)^2}(Q(V)+\hat{\Gamma}^T\nabla_V\Phi(V)f(V)-\hat{\Gamma}^T\nabla_V\Phi(V)C\nabla_V^T\Phi(V)\hat{\Gamma}/4)\\
&\quad+\Sigma(V,\hat{u}_V)\tfrac{\delta_2}{2}\nabla_V\Phi(V)gB^{-1}g^T(V)J_{1V}(V)
\end{aligned}\tag{16}
$$

with $\hat{\beta}=\nabla_V\Phi(V)f(V)-\nabla_V\Phi(V)C\nabla_V^T\Phi(V)\hat{\Gamma}/2$, $\delta_1>0$ and $\delta_2>0$ design constants, $J_{1V}(V)$ as defined previously, and the operator $\Sigma(V,\hat{u}_V)$ given by

$$\Sigma(V,\hat{u}_V)=\begin{cases}0 & \text{if}\ \ J_{1V}^T(V)\dot{V}=J_{1V}^T(V)(f(V)-gB^{-1}g^T\nabla_V^T\Phi(V)\hat{\Gamma}/2)\ <\ 0\\ 1 & \text{otherwise}\end{cases}$$

The first term in (16) minimizes (15) and has been derived using a normalized gradient descent scheme with the auxiliary HJB error defined as $E_{HJB}=(\hat{H}^*(V,\hat{\Gamma}))^2/2$. The second term in the OLA tuning law in (16) ensures that the system states remain bounded while the SOLA scheme learns the optimal cost function. The basis function is given by $\Phi(V)=[\nabla_VV\ \nabla_VV^2\ \nabla_VV^3\ \nabla_Vsin(V)\ \nabla_Vsin(2V)\ \nabla_Vtanh(V)\ \nabla_Vtanh(2V)]^T$. The SOLA-based HJB regulation and vertical tracking design for an unmanned helicopter is illustrated in Figure 1.

3.3 Kinematic Controller

The overall control objective for the helicopter UAV is to hover around a desired position $\rho_0=[x_0,y_0,z_0]^T$ with a desired heading while maintaining stable flight.

In this subsection, the terms required by the kinematic controller are derived. First of all, the desired translational velocity in the z- direction, v_{zd}, is found to ensure that the helicopter position converges to the desired position. To design the controller for the unmanned helicopter, the tracking error for position and velocity must first be defined. The position tracking error is given by

$$e_\rho=\rho_d-\rho\ \in\ \mathcal{Q}^a \tag{17}$$

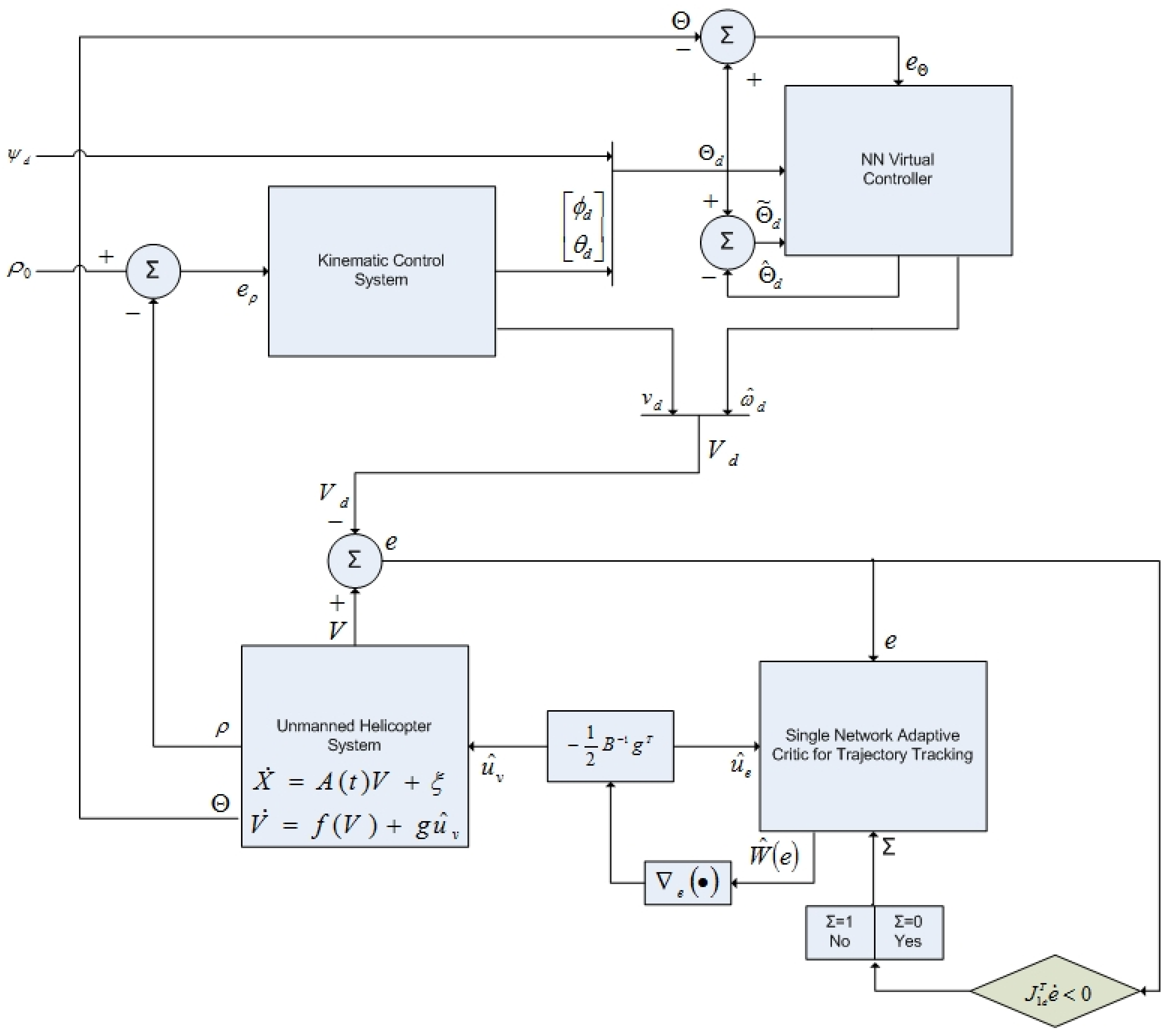

Figure 1. Control Scheme for Optimal Regulation of Helicopter

On differentiating (17) and substituting (1) from Dierks,[7] the position error dynamics are written as

$$\dot{e}_\rho = \dot{\rho}_d - Rv \tag{18}$$

In order to minimize the position tracking error, the desired velocity is defined as

$$v_d = [v_{dx}\ v_{dy}\ v_{dz}]^T = R^T(\dot{\rho}_d + K_\rho e_\rho) \ \in \ \mathcal{Q}^b \tag{19}$$

where $K_\rho = diag\{k_{\rho x}, k_{\rho y}, k_{\rho z}\} \ \in \ \mathbf{R}^{3\times 3}$ is a diagonal matrix with positive definite design constants. Next, define the translational velocity tracking error as $e_v = [e_{vx}\ e_{vy}\ e_{vz}]^T = [v_{dx}\ v_{dy}\ v_{dz}]^T - [v_x\ v_y\ v_z]^T = v_d - v$. By substituting v_d from (19) into (18) while observing $v \ = \ v_d - e_v$, the closed loop position error dynamics are written as $\dot{e}_\rho = -K_\rho e_\rho + Re_v$. Since $\dot{R}^T = -S(\omega)R^T$, one may obtain $\dot{v}_d = -S(\omega)v_d + R^T(\ddot{\rho}_d + K_\rho(\dot{\rho}_d - Rv))$, with $S(\omega)$ as given by Dierks.[7] Furthermore, it is important to note that there exist desired trajectories which may reach unstable operating regions of the helicopter as the orientation about the x- and y- axes approaches $\pm\pi/2$.

3.4 NN Virtual Controller

The main contribution of this subsection is the methodology for calculating the desired angular velocity. The desired roll, pitch, and yaw angles are compared with the actual orientation of the unmanned helicopter to get the attitude tracking error e_Θ. This information is sent as feedback to the virtual controller, which then calculates the desired angular velocity ω_d such that the orientation of the unmanned helicopter converges to the desired orientation (i.e., $\Theta \to \Theta_d$ or the attitude error e_Θ is minimized). For obtaining the desired angular velocity ω_d, the attitude tracking error is defined mathematically as $e_\Theta = \Theta_d - \Theta$. The attitude tracking error dynamics obtained using (3) in Dierks[7] are given by $\dot{e}_\Theta = \dot{\Theta}_d - T\omega$. The desired angular velocity ω_d, such that the orientation errors are zero, is given below

$$\omega_d = T^{-1}(\dot{\Theta}_d + K_\Theta e_\Theta) \tag{20}$$

where $K_\Theta = diag\{k_{\Theta 1}, k_{\Theta 2}, k_{\Theta 3}\} \ \in \ \mathbf{R}^{3\times 3}$ is a positive definite matrix with $k_{\Theta i} > 0$, $i = 1, 2, 3$. The angular velocity tracking error is defined as $e_\omega = \omega_d - \omega$. Since $\omega = \omega_d - e_\omega$, the closed-loop tracking orientation error dynamics can be written as $\dot{e}_\theta = -K_\theta e_\theta + Te_\omega$. To observe the dynamics of the proposed virtual controller, (20) is rearranged and written as

$$\begin{aligned} \dot{\Theta}_d &= T(\omega_d - T^{-1}K_\Theta e_\Theta) \\ \dot{\omega}_d &= \dot{T}^{-1}(\dot{\Theta}_d + K_\Theta e_\Theta) + T^{-1}(\ddot{\Theta}_d + K_\Theta \dot{e}_\Theta) \end{aligned} \tag{21}$$

For simplicity, a change of variables is defined such that $\Lambda_d \ = \ \omega_d - T^{-1}K_\Theta e_\Theta$, and (21) becomes

$$\begin{aligned} \dot{\Theta}_d &= T\Lambda_d \\ \dot{\Lambda}_d &= \dot{T}^{-1}\dot{\Theta}_d + T^{-1}\ddot{\Theta}_d = F_\Lambda \end{aligned} \tag{22}$$

Next, defining the estimates of Θ_d and Λ_d to be $\hat{\Theta}_d$ and $\hat{\Lambda}_d$ respectively, and the estimation error to be $\tilde{\Theta}_d = \Theta_d - \hat{\Theta}_d$, the dynamics of the virtual control inputs can be written as

$$\begin{aligned} \dot{\hat{\Theta}}_d &= T\hat{\Lambda}_d + K_{\Lambda 1}\tilde{\Theta}_d \\ \dot{\hat{\Lambda}}_d &= \hat{F}_{\Lambda 1} + K_{\Lambda 2}T^{-1}\tilde{\Theta}_d \end{aligned} \tag{23}$$

where $K_{\Lambda 1}$ and $K_{\Lambda 2}$ are positive constants. The estimate $\hat{\omega}_d$ can then be written as $\hat{\omega}_d = \hat{\Lambda}_d + T^{-1}K_\Theta e_\Theta + K_{\Lambda 3}T^{-1}\tilde{\Theta}_d$, where $K_{\Lambda 3}$ is another positive constant. Observing $\tilde{\omega}_d = \omega_d - \hat{\omega}_d \ = \ \tilde{\Lambda}_d - K_{\Lambda 3}T^{-1}\tilde{\Theta}_d$ and subtracting (23) from (22), as well as adding and subtracting $T^T\tilde{\Theta}_d$ and $K_{\Lambda 3}\dot{T}^{-1}\tilde{\Theta}_d$, the virtual controller estimation error dynamics are obtained as $\dot{\tilde{\Theta}}_d = T\tilde{\omega}_d - (K_{\Lambda 1} - K_{\Lambda 3})\tilde{\Theta}_d$ and $\dot{\tilde{\Lambda}}_d = (F_\Lambda + T^T\tilde{\Theta}_d - K_{\Lambda 3}\dot{T}^{-1}\tilde{\Theta}_d) - F_{\Lambda 1} -$

$K_{\Lambda 2}T^{-1}\tilde{\Theta}_d - T^T\tilde{\Theta}_d + K_{\Lambda 3}\dot{T}^{-1}\tilde{\Theta}_d$. In (23), the unknown function $F_{\Lambda 1}(x_\Lambda)$ is given by

$$F_{\Lambda 1}(x_\Lambda) = F_\Lambda + T^T\tilde{\Theta}_d - K_{\Lambda 3}\dot{T}^{-1}\tilde{\Theta}_d \tag{24}$$

The approximation properties of NN are utilized to estimate the unknown function $F_{\Lambda 1}(x_\Lambda) = W_\Lambda^T\sigma(V_\Lambda^T x_\Lambda) + \varepsilon_\Lambda$ by bounded target weights W_Λ^T, V_Λ^T such that $\|W\|_F \leq W_{K\Lambda}$, with $W_{K\Lambda}$ being a known constant, and ε_Λ the NN approximation error such that $\|\varepsilon_\Lambda\| \leq \varepsilon_{K\Lambda}$ for a constant $\varepsilon_{K\Lambda}$. The NN estimate of F_Λ is defined as

$$\hat{F}_\Lambda = \hat{W}_\Lambda^T\sigma(V_\Lambda^T\hat{x}_\Lambda) = \hat{W}_\Lambda^T\hat{\sigma}_\Lambda \tag{25}$$

where $\hat{W}_\Lambda^T$ is the NN estimate of W_Λ^T, and the NN input $\hat{x}_\Lambda = [1\ \rho_d^T\ \dot{\rho}_d^T\ \ddot{\rho}_d^T\ \dddot{\rho}_d^T\ \Theta_d^T\ \hat{\Lambda}_d^T\ V^T\ \tilde{\Theta}_d^T]^T$. The update law has been provided in [7], with F_Ω gains set to 20 and $\kappa_{\Omega 1} = 0.1$. On adding and subtracting $W_\Lambda^T\hat{\sigma}_\Lambda$ to the derivative of $\tilde{\omega}_d$, the estimation error dynamics for angular velocity are given by

$$\dot{\tilde{\omega}}_d = -K_{\Lambda 3}\tilde{\omega}_d + \tilde{F}_{\Lambda 1} - T^{-1}(K_{\Lambda 2} - K_{\Lambda 3}(K_{\Lambda 1} - K_{\Lambda 3}))\tilde{\Theta}_d - T^T\tilde{\Theta}_d + K_{\Lambda 3}\dot{T}^{-1}\tilde{\Theta}_d + \xi_\Lambda \tag{26}$$

where $\tilde{F}_{\Lambda 1} = \tilde{W}_{\Lambda 1}^T\hat{\sigma}_\Lambda$, $\tilde{W}_\Lambda^T = W_\Lambda^T - \hat{W}_\Lambda^T$, $\xi_\Lambda = \varepsilon_\Lambda + W_\Lambda^T\tilde{\sigma}_\Lambda$ and $\tilde{\sigma}_\Lambda = \sigma_\Lambda - \hat{\sigma}_\Lambda$. Also, it is to be noted that $\|\xi_\Lambda\| \leq \xi_{\Lambda K}$ with $\xi_{\Lambda K}$ being a positive constant defined as $\xi_{\Lambda K} = \varepsilon_{\Lambda K} + 2W_{K\Lambda}\sqrt{N_\Lambda}$, where N_Λ is the number of hidden layer neurons, and the relationship $\|\sigma_\Lambda\| \leq \sqrt{N_\Lambda}$ has been used.

3.5 NN Control Scheme Verification

In the final theorem, the stability of the entire system which includes position, orientation, and velocity tracking errors are considered along with the estimation errors of the virtual controller and the NN weight estimation errors. *Theorem 1* (Helicopter System Stability): Given the nonlinear helicopter UAV system defined in (4), with the target HJB equation (11), let the SOLA tuning law be given by (16). Let the auxiliary velocity control input be given by (12). Then the kinematic error, velocity tracking error, and NN parameter estimation errors of the cost function are all UUB for all $t \geq t_0 + T$, and the error system is regulated in a near optimal manner. That is, $\|u_e^* - \hat{u}_e\| \leq \varepsilon_u$ for a small positive constant ε_u. Theorem 1 can be shown to be valid. The theoretical results of Theorem 1 confirm that the estimation error remains bounded in the presence of bounded disturbances.

4. SIMULATION RESULTS

Simulation results for the unmanned helicopter are presented in this section. All simulations are performed in Simulink and demonstrate the performance of the proposed control scheme. The simulations take into account the aerodynamic features previously presented as part of the helicopter model. The following trajectories were used:

Case I Take-off and hovering:

$$z_d = 0.7$$

Case II Hovering and landing:

$$z_d = \begin{cases} 0.7 & t \leq 0.5\ sec \\ 0.7e^{0.5(t-10)} & otherwise \end{cases}$$

Note that the following gains and constants were used: $m = 44.3840kg$, $J = diag([1.4668\ 4.5767\ 4.4070]^T)kg.m^2$, $K_\Lambda = [24\ 80\ 20]^T$, $K_\rho = diag([1\ 1\ 3]^T)$, $K_\theta = diag([1\ 1\ 1]^T)$, and $\kappa = 0.1$. The virtual controller employs five hidden layer neurons, while the optimal controller employs seven hidden layer neurons. The optimal controller gains were set to $\delta_1 = 100$ and $\delta_2 = 1$. The helicopter's initial position and orientation are set to zero, and the requirement that $f(0) = 0$ is retained, except in the case of hovering followed by landing, in which case the initial value of z is set to the initial altitude. Figure 2 displays

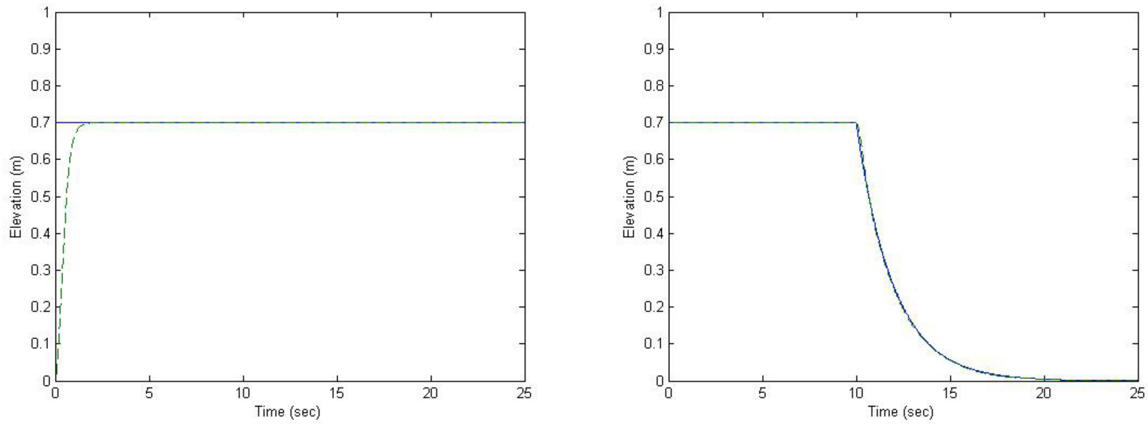

Figure 2. Hovering of Helicopter During Take-off (Left) and Landing (Right)

the regulation and vertical tracking capabilities of the helicopter when taking off and transitioning to hover, and when transitioning from hover to landing, respectively, with the previously detailed controls methodology. It is important to note that the main rotor thrust and tail rotor thrust should approach constant values rather than zero in order to keep the helicopter in hover, while the main rotor blade roll and pitch angles should approach zero.

5. CONCLUSION

A NN based optimal control law has been proposed for an unmanned helicopter which uses a single online approximator for optimal regulation and vertical tracking of an unmanned helicopter with dynamics written in strict-feedback form. The SOLA-based adaptive approach is designed to learn the infinite horizon continuous-time HJB equation, and the corresponding optimal control input that minimizes the HJB equation is calculated forward-in-time. Further, optimality of the controller has been demonstrated. A NN-based virtual control structure was used to obtain the desired angular velocities such that the desired orientation is achieved. Simulation results confirm that an unmanned helicopter with this control system is capable of regulated flight and vertical tracking.

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