

## DEVELOPMENT OF A MATHEMATICAL MODEL FOR SIMULATION OF COAL CRUSHING IN A HAMMER MILL

Avinash Tripathi, Ashish Agrawal and V.K. Gupta

Department of Fuel and Mineral Engineering  
Indian School of Mines, Dhanbad - 826 004, India

### ABSTRACT

Using the basic size-mass balance size reduction model for a continuous crushing operation and invoking the concept of a perfectly mixed system, a mathematical relationship has been obtained between the feed and product size distributions for a hammer mill. The model parameters 'breakage distribution function' for different size fractions of a Prime Coking Coal and an imported coal were determined by conducting appropriate tests in a laboratory hammer mill. Using data generated in the Rourkela Steel Plant on a Production hammer mill, variation of the second set of model parameters, 'absolute rate of breakage of particles of each size class', with coal feed rate to the mill was established. Analysis of the data generated in the plant has shown that only +10 mm particles of the imported coal broke faster than the PCC particles of the same size. The developed mathematical model can be used to stimulate the performance of the crushing plant in respect of the effect of feed size distribution and feed rate, and for taking control action for keeping the product fineness constant by adjusting the coal feed rate to the hammer mill.

**Keywords:** Hammer mill, Coal crushing, Mathematical model, Size-mass balance, Product size distribution control.

### INTRODUCTION

At steel plants, for production of coke in the coke ovens the coal received from coal mines is crushed to a desired degree of fineness. Size distribution of the coal charged to the coke ovens has a significant effect on the quality of the coke produced as both the oversize and undersize particles are detrimental to coke quality. Thus, control of the coal crushing operation is a task of considerable importance. In India, generally hammer mills are used for crushing of coal. And, the size distribution of the crushed coal is specified as: % passing 3 mm (80–82%) and % passing 0.5 mm (30–35%). Determination of the optimum values of the operating parameters through simulation of the process performance with the help of a mathematical model is now a well established practice. In view of these facts, a mathematical model is presented in this paper for simulating crushing of coal in hammer mills.

### MODEL DEVELOPMENT

Breakage kinetics in most mills can be described by a first order reaction rate expression:<sup>[1, 2]</sup>

$$dh_i / dt = -r_i^* h_i + b_{i,i}^* r_i^* h_i \quad \dots (1)$$

where,

$r_i^*$  : breakage rate constant for particles of size class i.

t : time

$h_i$  : weight fraction of mill holdup of solids (coal) in the sieve size interval i at any given time t.

$b_{i,i}^*$  : breakage distribution parameter, weight fraction of particles of size class i that reports to size class i after undergoing breakage.

For convenience, let us define a modified breakage distribution function,  $b_{i,j}$ , known as practical breakage distribution function in the literature, as:<sup>[3]</sup>

$$b_{i,j} = b_{i,j}^* / (1 - b_{j,j}^*) \quad \dots (2)$$

where,

$b_{i,j}$ : modified breakage distribution parameter-weight fraction of particles leaving size class j that reports to size class i after undergoing breakage.

$b_{i,j}^*$  : breakage distribution parameter- weight fraction of particles of size class j that reports to size class i after undergoing breakage.

The corresponding breakage rate constant is defined as:

$$r_j = r_j^* (1 - b_{j,j}^*) \quad \dots (3)$$

where  $r_j$  is modified breakage rate constant for particles of size class j,  $hr^{-1}$ . Thus,

$$r_j^* b_{i,j}^* = r_j b_{i,j} \quad \dots (4)$$

For a steady state operation we can write:

Rate of entry – Rate of breakage – Rate of Production – Rate of exit = 0,

$$Ff_i - Fh_i - Hr_i h_i + \sum_{j=1}^{i-1} Hr_j b_{i,j} h_j = 0 \quad \dots (5)$$

where,

F : feed rate of coal to hammer mill, t/hr.

H : total weight of coal particles in the mill.

$f_i$  : weight fractions of size i particles in the feed to the hammer mill.

Noting that for a fully mixed mill, as is the case with a hammer mill, we have

$$h_i = p_i \quad \dots (6)$$

$$Ff_i - Fp_i - Hr_i p_i + \sum_{j=1}^{i-1} Hr_j b_{i,j} p_j = 0 \quad \dots (7)$$

where  $p_i$  is weight fraction of particles of size class i in the mill discharge (product).

Rearranging we obtain,

$$p_i = \frac{f_i + \sum_{j=1}^{i-1} (f_j - p_j) b_{i,j}}{1 + r_i \tau} \quad \dots (8)$$

where,  $\tau = H/F$ , is known as the mean residence time of coal particles in the mill.

For simulation purposes, the task can now be defined as: 'quantification of the effect of feed rate on the mathematical model parameters  $b$  and  $r$ '. As shown by a number of researchers,<sup>[4]</sup> generally  $b$  parameters are not affected appreciably by the feed rate (or more correctly, the hold up of the solids in the mill, which increases with feed rate). Further, it should be pointed out that it was not possible to measure the mill hold-up weight of coal, only the product  $r_i H$  or  $r_i \tau$  could be estimated (i.e. each  $r_i$  value multiplied by a constant  $H$  or  $\tau$ ), not the  $r_i$  value alone. However, the term  $r_i H$  has the special significance as it represents the absolute rate of breakage of size  $i$  particles in the mill in tons/hour.<sup>[5, 6]</sup> For these two joint parameters we have the following relationships:

$$r_i \tau = \frac{(f_i - p_i) + \sum_{j=1}^{i-1} (f_j - p_j) b_{i,j}}{p_i} \quad \dots (9)$$

or,

$$r_i H = \frac{F}{p_i} \left[ (f_i - p_i) + \sum_{j=1}^{i-1} (f_j - p_j) b_{i,j} \right] \quad \dots (10)$$

## EXPERIMENTAL

### Plant tests

Tests were conducted on one of the two secondary hammer mills of the million ton coal handling facility of the Rourkela Steel Plant.<sup>[7]</sup> Samples of the mill feed and discharge were collected only during those periods when plant was observed to be running under near steady state conditions (as reflected from the load and the size distribution of coal on the conveyor carrying the mill discharge). For better accuracy in the results, samples collection was done at two locations on the conveyor. Depending on the feed rate, the weight of each test sample varied from 20 to 30 kg. The weights of the feed and discharge samples were found to be nearly same as would be expected under steady state conditions. In some cases two samples of feed were taken—one before taking the discharge sample and the other afterwards. The difference in the size distributions of two samples was found to be practically negligible. In this way, it was ensured that near steady state conditions prevailed during the tests.

For the Prime Coking Coal (PCC), four sets of the feed and product (discharge) size distribution data could be generated corresponding to feed rates: 73, 93, 160 and 269 t/hr. Data on Imported coal corresponded to 195 and 224 t/hr.

**Laboratory experiments**

To determine the breakage distribution function for two types of coal, experiments were carried out in a laboratory hammer mill (12 inch diameter × 3in. width) As the mill had big rectangular slots all along its periphery, it was argued that when fed slowly, feed particles will undergo breakage only once during their passage through the mill. Thus, if only one selected size fraction of coal is fed to the hammer mill, the mill discharge size distribution should correspond to the breakage distribution function,  $b^*$ , of this size fraction.

For this purpose, test samples of crushed PCC and imported coal were screened to obtain +10, 6/10, 3/6 and 1/3 mm size fractions. About 400 g of each size fraction was slowly fed to the mill separately. The mill discharge thus obtained was screened using the same series of screens.  $b^*$  values could be calculated from this data as discussed above.

**RESULTS**

Breakage distribution functions,  $b_{i,j}$ , for PCC and Imported coals as obtained from the laboratory hammer mill test results and eq. (2) are given in Tables 1 and 2.

Variation of  $r_i H$  with feed rate is shown in Fig.1 for all the four size classes under consideration: (1) + 10 mm, (2) 6–10 mm, (3) 3–6 mm, and (4) 1–3 mm. It will be seen that depending on the particle size, the absolute rate of breakage initially either decreases (for +10 mm particles) or increases ( for the remaining three finer size fractions) with increase in the feed rate, and beyond about 175 t/hr feed rate there is practically no change, i.e.

$$r_i H = K_i, F > 175 \text{ t/hr} \quad \dots (11)$$

where  $K_i$  is a mill-material constant for size fraction  $i$ .

Table 1: Breakage distribution matrix,  $b_{i,j}$ , for PCC Coal

Size interval $i$ (mm)	$b_{i,j}$				
	Feed size $j$	+ 10	6–10	3–6	1–3
+ 10		0.000	0.000	0.000	0.000
6–10		0.005	0.000	0.000	0.000
3–6		0.056	0.048	0.000	0.000
1–3		0.281	0.276	0.315	0.000
–1		0.658	0.676	0.685	1.000

Table 2: Breakage distribution matrix,  $b_{i,j}$ , for imported coal

Size interval $i$ (mm)	$b_{i,j}$				
	Feed size $j$	+ 10	6–10	3–6	1–3
+ 10		0.000	0.000	0.000	0.000
6–10		0.000	0.000	0.000	0.000
3–6		0.030	0.030	0.000	0.000
1–3		0.224	0.216	0.235	0.000
–1		0.746	0.754	0.765	1.000

As can be seen in eq. (8), it is the parameter  $r_i \tau$  that directly relates  $p_i$  to  $f_i$ . Especially, in the case of top size (+10 mm) and the second size (for which  $b_{2,1} = 0$ ) eq. (8) simplifies to,

$$p_1 = f_1 / (1 + r_1 \tau) \quad \dots (12)$$

$$p_2 = f_2 / (1 + r_2 \tau) \quad \dots (13)$$

This means that a fraction  $1/(1 + r_1 \tau)$  of feed of size class 1 remains unbroken. And, similarly a fraction  $1/(1 + r_2 \tau)$  of size class 2 remains unbroken.

Figure 2 shows variation of  $r_i \tau$  with feed rate for the four size fractions under consideration. It will be seen that: (i)  $r_1 \tau$  with feed rate quite sharply (sharper than the rate of decline observed in case of  $r_1 H$ ), (ii)  $r_2 \tau$  decreases with increase in feed rate, though at a much slower rate than that observed in case of  $r_1 \tau$  (while  $r_2 H$  was observed to increase with feed rate), (iii)  $r_3 \tau$  varies little feed rate, and (iv)  $r_4 \tau$  increases with feed rate (but at a slower rate than that observed in case of  $r_4 H$ ).

These results show that +10 mm particles are broken much more effectively at lower feed rates. Particles in the size class 6–10 mm also break better at lower feed rate, though the feed rate effect is comparatively less pronounced. Breakage of particles of size class 3–6 mm is not affected by feed rate. And, particles in the size range 1–3 mm break better at higher feed rates (i.e. when hammer mill is loaded with more material, because mill hold-up of solids is expected to increase with feed rate).

An inspection of curves shown in Figures 1 and 2 shows that it is not possible to describe these curves in terms of simple mathematical functions. The best that can be done is to describe  $r_i H$  curves piecewise, in two parts. The first part corresponds to low values of feed rate. This part is not of practical interest. The second part corresponds to nearly constant absolute rate of breakage, i.e.  $r_i H = K_i$ . As  $r_i \tau = r_i H / F$ , corresponding expressions for  $r_i \tau$  follows immediately from eq. (11).

It can be seen in Figs. 1 and 2 that +10 mm particles of imported coal break much faster rate than the same size PCC particles. However, the remaining three finer size fractions of imported coal break at practically the same rate as the PCC particles of these sizes. This is an important point to be noted.

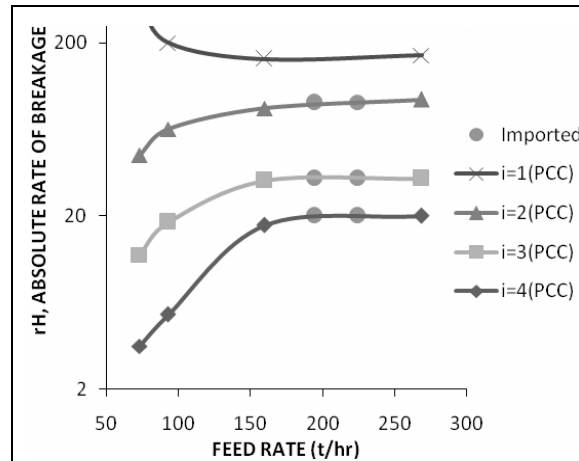


Fig. 1: Variation in the absolute rate of breakage '  $r_i H$ ,' with feed rate for different size of coal.

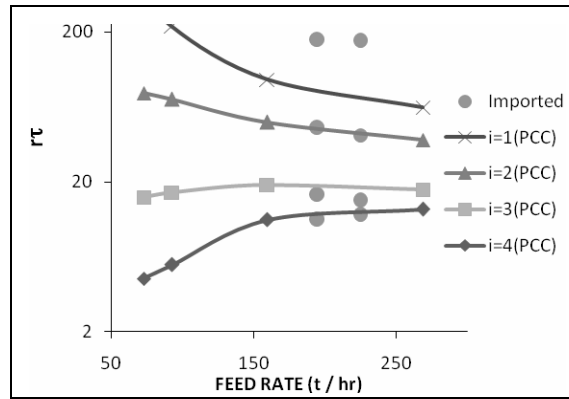


Fig. 2: Variation in the parameter ' $r_i\tau$ ' with feed rate for different size fractions of coal.

## DISCUSSION

With a view to determining the optimum feed rate to get the desired size distribution of 80% passing 3 mm screen, some simulation experiments were carried out using the developed mathematical model. Two feed size distributions were selected: (i) Feed-1 corresponding to the average feed observed for PCC and (ii) Feed-2 corresponding to the average feed observed for Imported coal. Using appropriate equations, mill discharge size distributions were calculated. It was observed that for a typical PCC feed (Feed-1 size distribution) the wt. % passing 3 mm screen decreases from 69.2 to 62.2 as the feed tonnage was increased from 75 to 225 t/hr. Even at 75 t/hr feed rate, however, the discharge was not fine enough as the wt.% passing 3 mm screen was only 69.2. It was observed that if somehow feed to the secondary hammer mill is made as fine as feed-2, the wt.% passing 3 mm screen can be as high as 79.4 at 75 t/hr feed rate. Moreover, even if feed rate is increased to 225 t/hr, this percentage reduces only marginally to 76%. In this context it is interesting to point out that the mill discharge at the feed rate of 75–125 t/hr for feed-1 has practically the same size distribution as feed-2. This observation suggests that in order to get the product of desired size distribution either a tertiary hammer mill should be used or both the primary and secondary mills should be made more effective through suitable modifications in their design.

It was observed that in case of Imported coal the desired size distribution is achieved to a reasonable degree of closeness even when the feed rate is as high as 275 t/hr.

## CONCLUSION

1. By carrying out a dynamic size-mass balance around the hammer mill and by invoking the concept of 'perfectly mixed system', a mathematical relationship has been obtained between the mill feed and product size distributions.
2. Using the actual plant data, variation of the model parameters  $r_iH$  (absolute rate of breakage of particles of size class  $i$  in the hammer mill) with feed rate has been established. Thus, it has now become possible to simulate the performance of the crushing plant in respect of the effect of feed size distribution and feed rate.
3. At about 200 t/hr feed rate the value of these parameters for 6–10, 3–6 and 1–3 mm size fractions of Imported coal were found to be practically the same as those obtained for PCC.

However, for the +10 mm size fraction of Imported coal the absolute rate of breakage was found to be about 2.2 times higher than, that observed in the case of PCC. These results indicate that only coarse +10 mm particles of Imported coal break faster than the same size PCC particles.

4. The tests carried out in a small laboratory hammer mill for establishing breakage distribution functions have shown that all coarse size fractions of Imported coal produce about 78% –1 mm material as they undergo breakage in the hammer mill. As expected, this percentage value is lower for PCC (72%).
5. The degree of reduction (elimination due to breakage) for particles smaller than 10 mm was found to be very low (9–30 % only). To break these particles more effectively, the design of the hammer mill needs to be improved.

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