

Mathematical model for predicting solidification and cooling of steel inside mould and in air

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ABSTRACT

A two-dimensional mathematical model has been developed to describe the solidification and cooling of steel inside the mould after teeming and in the air after stripping. Partial differential equations describing the processes have been discretized using control volume approach. The discretization equations obtained are of Tri-diagonal matrix form, which have been solved using well known Tri-diagonal matrix algorithm (TDMA) and Alternate direction implicit (ADI) solver. The model has been validated by measuring surface temperatures of mould and ingot using Infrared thermo-vision scanner. This is then used to compute charging temperature and solidification status of ingot as function of track time and type of ingot.

INTRODUCTION

The process of cooling and solidification of liquid steel plays an important role in steelmaking industry. Liquid steel made in LD converters, open hearth furnaces or electric arc furnaces are poured into (i) ingot moulds or (ii) a continuous casting systems, which involve the cooling and solidification processes. In this paper, a mould for the former case with a particular reference to Rourkela Steel Plant (RSP) has been described. In this plant, steel is being poured into reverse-tapered moulds from the top. The detail of various sizes of mould/ingot under use at RSP is shown in Table - 1.

Table - 1 : Details of Various sizes of mould/ingot

Type of mould	Weight of ingot (ton)	Height of mould (mm)	Top Surface			Bottom Surface		
			Length	Width	Thickness	Length	Width	Thickness
				(mm)			(mm)	
8.33 t	8.33	2200	1030	510	165	1100	600	175
10 t	10	2320	1356	534	175	1420	600	175
B type	11	2200	1198	614	175	1262	680	185
C type	12.5	2350	1394	569	175	1460	635	185
D type	16	2350	1159	840	175	1230	909	185
E type	16.8	2350	1536	676	175	1600	740	185
18 t	18	2350	1280	850	175	1350	920	185

DESCRIPTION OF MATHEMATICAL MODEL

Assumptions

1. It is a two-dimensional model, in which effect of variation of temperature along the ingot height is neglected.
2. During solidification, each grid known as zone henceforth is assumed to undergo independent solidification as a function of temperature. There is heat transfer but no mass transfer taking place across the zone boundaries.
3. Effect of convection current in the liquid pool during solidification is neglected.
4. The ingot and mould are considered rectangular in cross-section.

Formation of air gap (period between teeming and stripping)

While teeming, ingot and mould are in contact with each other but with lapse of time an air gap is formed between the ingot and the mould because of shrinkage of steel in the process of solidification. Air gap can be influenced by factors like ingot size and shape, composition of steel, mould type and teeming temperature.

The gap is first formed at the bottom of the ingot and then propagated upwards. The effect of time of gap formation on the internal temperature distribution has been found by Sarjant, et. al.^[1]. They also found that air gap formation has no significant effect on the rate of

advance of solidification front. Air gap formation increases the time of completeness of solidification.

Solidification process

The solidification process involving a complex heat and mass transfer phenomenon has been a subject of great interest to researchers and academicians. The solution of problem starts with classical Stefan problem for phase changes in pure substance, which can be solved analytically. The process of solidification of a metal containing a small concentration of impurity demands numerical technique. Crowley and Ockendon^[2] have reported a model for solidification of dilute alloy in which equation of heat and mass transfer are coupled through the conditions at the moving phase boundary. The temperature of melt in each zone drops on account of heat transfer to the adjoining zones and finally to the surrounding (mould or ambient) through the ingot surface. When this temperature falls below liquidus, solid phase appears, the amount of which can be determined by Lever's rule of phase change^[3].

Governing equation

The governing equation describing the thermal history of the ingots from teeming to charging is as follows :

$$\rho c(\partial T/\partial t) = \partial/\partial x (k \partial T/\partial x) + \partial/\partial y (k \partial T/\partial y) + q \quad \dots 1$$

where q is the source term representing the latent heat evolved during solidification. Expression for q can be written as :

$$q = \rho L \left(\frac{\partial \Psi}{\partial t} \cdot \frac{\partial T}{\partial t} \right) \quad \dots 2$$

Rate of solidification

Steel is a solid solution of iron and elements like carbon, silicon, manganese, etc. Composition of steel determines liquids/solidus temperatures. Hence for a given heat removal rate, solidification time will depend upon concentration of carbon, silicon, etc. in steel. Effect of carbon concentration is predominant over other elements enabling us to treat steel as a binary alloy of iron and carbon.

Referring to iron-carbon diagram as shown in Fig. 1 and applying phase rule, following expression can be obtained

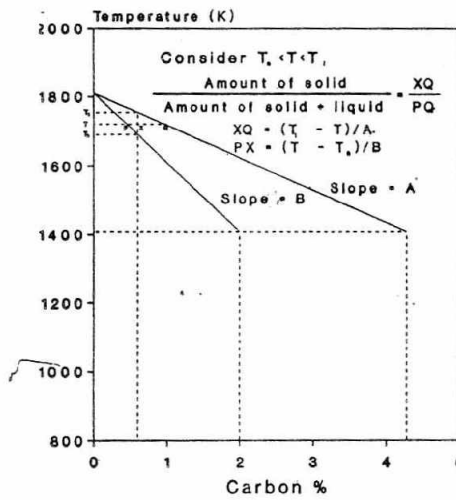


Fig. 1 : Iron-carbon diagram

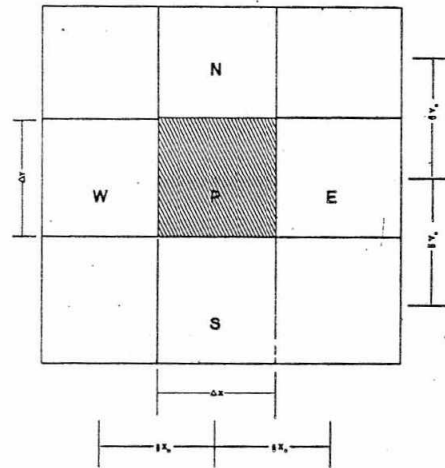


Fig. 2 : Grid spacing

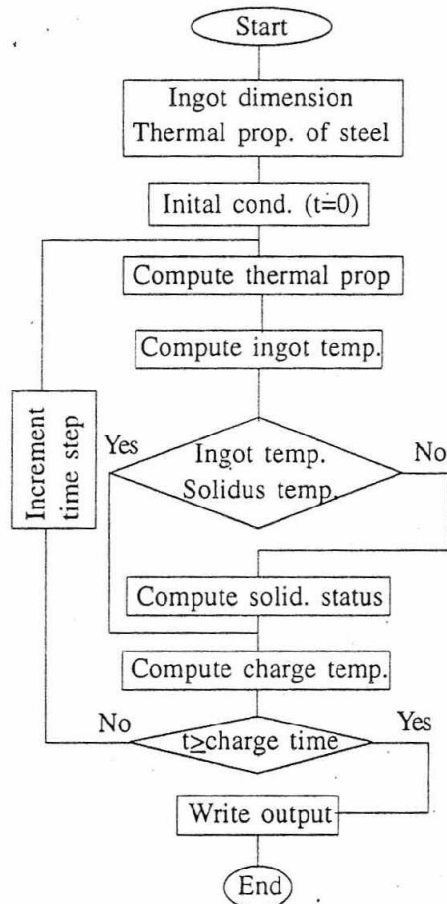


Fig. 3 : Flow char for cooling/solidification model

$$\partial\Psi/\partial T = [A'B'(T_s - T_l)(1 - \delta\Psi)/\{A'(T - T_s) + B'(T_l - T)\}^2] \quad \dots 3$$

Expressions for solidus and liquidus temperatures as function of carbon concentration can be written as:

$$T_s = T_{Fe} - B'C_c \quad \dots 4$$

$$T_l = T_{Fe} - A'C_c \quad \dots 5$$

Initial condition

(a) For ingot

$$T = T_p \text{ (Internal grids) and } = T_s \text{ (Boundary grids)} \quad \dots 6$$

(b) For Mould

$$T_\zeta = T_{mo} + (T_{mi} - T_{mo})(\zeta/d_m)^5 \quad \dots 7$$

where T_ζ is temperature in mould interior point at distance ζ measured from outer surface of mould.

Boundary condition

i) For the period teeming to stripping

(a) *Outer boundary condition*

Heat transmission by radiation and convection between mould outer surface and atmosphere

$$-k[\partial T/\partial n]_e = \sigma\epsilon_m(T_{mo}^4 - T_\infty^4) + h_{m\infty}(T_{mo} - T_\infty) \quad \dots 8$$

(b) *Ingot-mould interfacial boundary condition*

Heat transmission by conduction and radiation between mould inner surface and ingot surface within the air gap

$$-k[\partial T/\partial n]_e = k_a(T_{io} - T_{mi})d_a + \sigma\epsilon_a(T_{io}^4 - T_{mi}^4) \quad \dots 9$$

ii) For the period stripping to charging

Heat transmission by radiation and convection between ingot outer surface and atmosphere

$$-k[\partial T/\partial n]_e = \sigma\epsilon_i(T_{io}^4 - T_\infty^4) + h_{i\infty}(T_{io} - T_\infty) \quad \dots 10$$

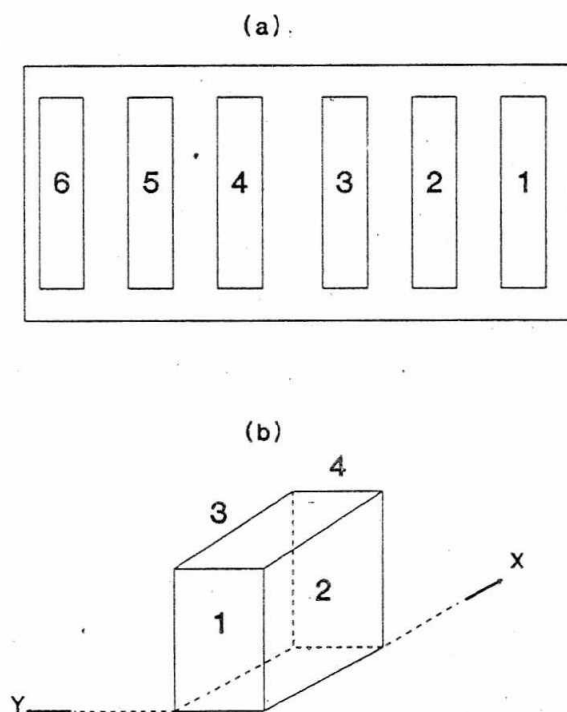


Fig. 4 : Ingot arrangement in mould-train (a)
and mould/ingot surface nomenclature (b)

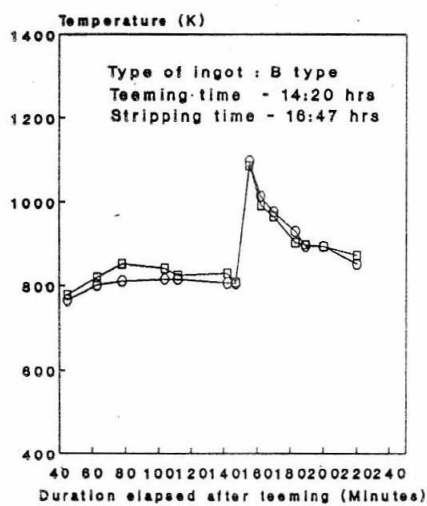


Fig. 5: Comparison of experimental
and predicted temperature during
cooling of ingot

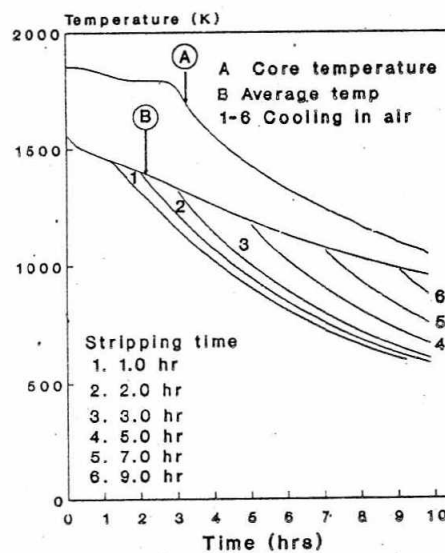


Fig. 6: Cooling of B type ingot in
mould and air

METHODOLOGY OF SOLUTION

Discretization Equation

Using control volume approach ^[4], the following discretization equation is obtained with respect to equation (1).

$$a_p T_p = a_e T_e + a_w T_w + a_n T_n + a_s T_s + (a_p^o - S_c) T_p^o + C^* \quad \dots 11$$

where

$$a_e = k_e \Delta y / \delta x_e \quad \dots 12$$

$$a_w = k_w \Delta y / \delta x_w \quad \dots 13$$

$$a_n = k_n \Delta x / \delta y_n \quad \dots 14$$

$$a_s = k_s \Delta x / \delta y_s \quad \dots 15$$

$$a_p^o = \rho c \Delta x \Delta y / \Delta t \quad \dots 16$$

$$S_c = \rho L \Delta x \Delta y (\partial \psi / \partial T) \Delta t \quad \dots 17$$

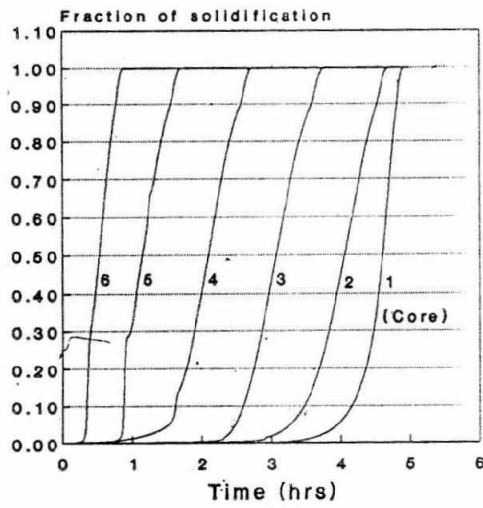
$$C^* = 0 \text{ for internal grids} \quad \dots 18$$

$$a_p = a_e + a_w + a_n + a_s + a_p^o - S_c \quad \dots 19$$

Definitions of Δx , Δy , δx_e , δx_w , δy_n , δy_s are shown in Fig. 2. The interfacial thermal conductivities k_e , k_w , k_n , k_s are defined as harmonic mean of thermal conductivities of two adjacent grid points across the interface. Source term S_c will be absent in case of completely solidified ingot and for grids lying in the mould portion.

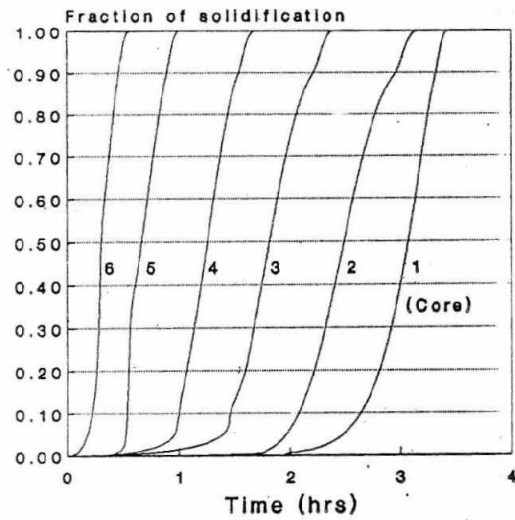
Solution of Discretization Equation

Alternate direction implicit (ADI) solver has been employed for the solution of the discretization equation ^[11]. In this method, boundary condition information from the ends of line is transmitted at once to the interior of the domain, irrespective of grid sizes. Thus faster convergence is ensured. The discretization equation is of the Tridiagonal



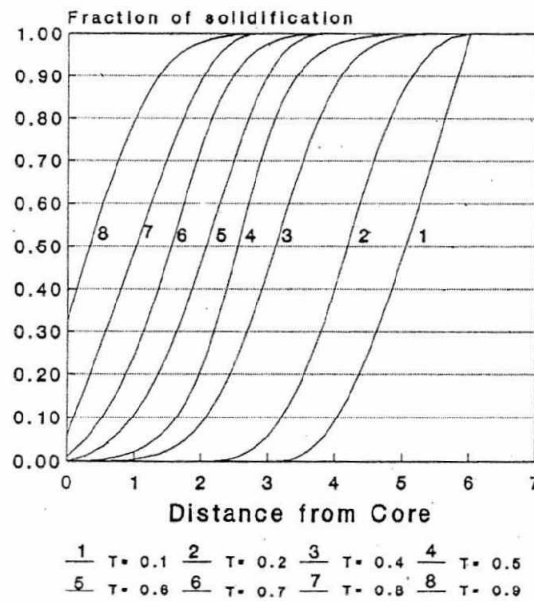
$\frac{1}{4}$ (12,12)grid $\frac{2}{5}$ (11,12)grid $\frac{3}{6}$ (10,12)grid
 $\frac{4}{7}$ (9,12)grid $\frac{5}{8}$ (8,12)grid $\frac{6}{9}$ (7,12)grid

Fig. 7 : Solidification rate for D type ingot inside mould



$\frac{1}{4}$ (12,12)grid $\frac{2}{5}$ (11,12)grid $\frac{3}{6}$ (10,12)grid
 $\frac{4}{7}$ (9,12)grid $\frac{5}{8}$ (8,12)grid $\frac{6}{9}$ (7,12)grid

Fig. 8 : Solidification rate for B type ingot inside mould



$\frac{1}{5}$ T = 0.1 $\frac{2}{6}$ T = 0.2 $\frac{3}{7}$ T = 0.4 $\frac{4}{8}$ T = 0.5
 $\frac{5}{9}$ T = 0.6 $\frac{6}{10}$ T = 0.7 $\frac{7}{11}$ T = 0.8 $\frac{8}{12}$ T = 0.9

Fig. 9 : Movement of solidification front for D type ingot inside mould

matrix form. Hence a well known Tridiagonal matrix algorithm (TDMA) has been employed^[5]. The computer program written in FORTRAN 77 can run in PC (80386 or higher version). A macro flowchart describing the program is shown in Fig. 3.

VALIDATION OF MODEL

The surface temperatures of mould an ingot were measured using AGA. Thermovision 750 instrument. The movement of ingot was followed from teeming bay to soaking pit. The ingot arrangement in mould train and nomenclature for surface identification of mould/ingot is shown in Fig. 4. The cooling model was run to adjust the heat transfer coefficient suitably so that the predicted and experimental results match. Fig. 5 represents a comparison of theoretical and experimental results for B type of ingot.

CALCULATION OF CHARGING TEMPERATURE

Tables 2 to 3 show the charging temperatures calculated for various combinations of stripping and charging times for 'B' and 'D' type ingots. A comparative study of tables reveals that 'B' type of ingot cools faster than 'D' type, which is due to variation of mould thickness. Fig. 6 shows the cooling of 'B' type ingot in mould and air after it is teemed. The upper curve shows the decrease of core temperature in the mould, whereas the lower curves show the variation of average temperature. It is evident from the figure that during initial cooling of ingot in the mould its average temperature drops more rapidly as compared to core temperature. The core of ingot cools very slowly on account of evolution of latent heat of solidification in the liquid core. After complete solidification of ingot, both average and core temperatures drop rapidly when stripped from the mould. The visible link in the core temperature may be attributed to the phase transformation taking place in the steel.

A comparative study of the curves tells that the average temperature of an ingot stripped 60 minutes after teeming and charged 60 min. after stripping is the same as average temperature of an ingot stripped after 3.6 hrs. It may also be seen from the figure that the core remains unsolidified for about 3.4 hrs. The unsolidified portion is a source of potential heat and hence ingot with track time less than 1.5 hrs. would require minimum fuel for heating them into the soaking pit to rollable temperature.

Table 2 : Charging temperature for 'B' type ingot (K)

Time in mould (hrs.)	Time elapsed after stripping (hrs.)									
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
0.033	1611	1466	1284	1115	970	851	751	667	600	547
1.0	1515	1318	1145	994	870	766	678	609	553	509
2.0	1446	1229	1061	925	813	717	640	578	529	489
3.0	1372	1148	993	868	765	677	608	553	508	472
4.0	1289	1078	936	821	723	645	582	532	491	459
5.0	1215	1018	886	778	688	616	560	514	477	447
6.0	1147	965	842	740	658	592	540	498	464	436
7.0	1086	917	802	706	631	571	523	484	453	427
8.0	1032	873	769	676	607	552	508	472	443	419
9.0	983	835	731	650	586	535	494	461	434	411
10.0	938	800	703	627	568	521	482	451	425	405

Table 3 : Charging temperature for 'D' type ingot (K)

Time in mould (hrs.)	Time elapsed after stripping (hrs.)									
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
0.033	1614	1505	1346	1214	1098	975	876	790	712	646
1.0	1533	1358	1223	1096	979	881	794	716	649	594
2.0	1454	1286	1154	1030	924	831	750	677	617	568
3.0	1390	1223	1089	974	874	788	711	645	591	546
4.0	1328	1161	1032	923	830	748	676	617	567	526
5.0	1265	1102	980	878	791	713	647	592	547	509
6.0	1202	1048	933	838	754	679	621	571	529	494
7.0	1146	1000	891	801	721	653	598	552	513	481
8.0	1094	956	853	767	691	629	577	535	499	469
9.0	1047	915	818	735	665	607	559	519	486	458
10.0	1004	878	785	706	641	587	543	506	475	449

CALCULATION OF SOLIDIFICATION STATUS

Table 4 shows solidification times for various sizes of ingot for two cases, namely (i) ingot remaining inside the mould and (ii) ingot being stripped after 1.0 hour. As it can be seen that solidification time increases with increase in size of ingot. This inference is in conformity with earlier work. Cooling becomes faster after stripping to the tune of 0.4 - 1.167 hours for the case under consideration. Fraction of solidification for various zones including core with time have shown in Figs. 7 & 8 for 'D' and 'B' type ingot respectively. Fig. 9 represents movement of solidification front for 'D' type ingot.

Table 4 : Solidification time for various sizes of ingot

Sl. No.	Type of ingot	Ingot dimension in mm (length x width)	Solidification time (hour)	
			Inside mould	After 1.0 hour Stripping
1	8.33 t	1077 x 570	2.600	2.200
2.	B' type	1241 x 658	3.433	2.733
3.	C type	1438 x 613	3.067	2.467
4.	D type	1206 x 886	4.9.33	3.867
5.	E type	1579 x 719	4.000	3.067
6.	18 t	1327 x 897	5.233	4.067

CONCLUSION

The model is capable of computing ingot temperature distribution and solidification status after the steel is teemed and till it is charged inside the soaking pit. This enables the calculation of the charging temperature of ingot as a function of track time and type of ingot. This model provides a tool for off-line calculation of initial condition for off-line/on-line heating model of soaking pit.

NOMENCLATURE

- A' = Slope of liquidus
- B' = Slope of solidus
- C_c = Concentration of carbon, %
- c = Specific heat, kJ/(kg K)
- d = Thickness of mould, m
- h = Convective heat transfer coefficient, W/(m²K)

K	=	Thermal conductivity, W/(mK)
L	=	Latent heat of solidification, kJ/kg
n	=	Direction of heat conduction
q	=	Rate of heat generation per unit volume, W/m ³
T	=	Temperature, K
T _{Fe}	=	Melting temperature of pure iron, K
T _p	=	Pouring temperature of steel, K
t	=	Time, hr
Δt	=	Time step, hr
x	=	X-co-ordinate
y	=	Y-co-ordinate
ε	=	Emissivity
ρ	=	Density, kg/m ³
σ	=	Stefan-Boltzmann constant
ψ	=	Volume of solid phase per unit total volume constituting solid and liquid
ζ	=	Distance measured inside mould from its outer surface
δ	=	Infinitesimal value

Subscripts and superscripts

a	=	Air entrapped inside gap between mould and ingot
e	=	Surface
i	=	Ingot; inner surface
l	=	Liquidus
m	=	Mould
o	=	Outer surface
s	=	Solidus
∞	=	Ambient

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