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COMPLIANCE CRACK LENGTH RELATIONS FOR THE FOUR-POINT BEND SPECIMEN

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Abstract—Compliance crack length relations for the four-point bend specimen geometry have not been reported in the literature in spite of this geometry being one of the popularly used specimens for fatigue crack growth studies. An effort has been made in the present work to fill this gap. Accordingly, the finite element technique was employed to simulate loading and calculate displacements at various locations in a four-point bend specimen. The load–displacement data thus obtained were processed to yield compliance crack length relations. These relations were employed to calculate the crack length during fatigue testing of four-point bend specimens in which the crack length was also measured by optical means. A good correlation was observed between the predicted crack length and that measured optically.

INTRODUCTION

When fatigue crack propagation data are generated in the laboratory, the accurate measurement of crack length is of vital importance in ensuring the reliability of the data. Crack lengths can be measured by direct and indirect means. While direct methods of crack length measurement, e.g. by travelling microscope, are tedious and prone to human error, the indirect methods are not only superior in these respects but are also amenable to automation and therefore useful for computer-controlled fatigue testing. One of the indirect methods of measuring crack length that is popularly used is the compliance technique. The elastic compliance of a specimen, defined as the displacement per unit load (V/P), increases with the length of the crack contained in it, and this fact forms the basis of the technique. For practical use and especially for computerized measurement of crack lengths, compliance crack length (CCL) relations, expressed as

$$\frac{a}{W} = f\left(\frac{EBV}{P}\right), \tag{1}$$

where a is the crack length, W is the width of the specimen, E is the Young's modulus, B is the specimen thickness and f is a polynomial function, are employed. The parameters W, E and B in eq. (1) are used to normalize the variables so that the equation becomes independent of specimen size and material. Saxena and Hudak [1] have proposed that CCL relations be written in the form

$$\frac{a}{W} = f(u), \tag{2}$$

where

$$u = \frac{1}{\sqrt{\left(\frac{EBV}{P}\right)} + 1}. \tag{3}$$

Such a form has been found to facilitate the derivation of the polynomial function f and has now been adopted universally. CCL relations are specific to specimen geometry and the measurement location of the displacement, V. Hence, for the various configurations possible they have to be developed individually.

While CCL relations, of the form given in eq. (2), have been developed for most specimen geometries [1–5], the four-point bend (4PB) specimen has not received attention in this regard. The 4PB specimen is one of the most popular configurations used for fatigue crack growth testing. It provides a central region in which the bending moment is uniform, and is therefore often preferred

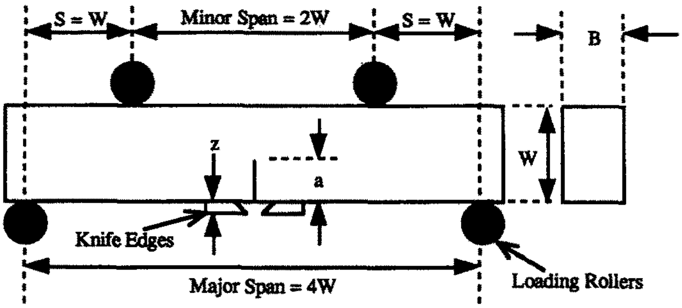

Fig. 1. The 4PB specimen geometry and loading configuration.

to the three-point bend geometry. Hence, in this work, CCL relations have been developed for the 4PB geometry by employing the finite element method, and subsequently verified experimentally.

During fatigue crack growth testing, displacements are customarily monitored either at the crack mouth or at knife-edges, which are external to the geometry of the specimen and which may vary in size. These displacements are processed numerically to compute the compliance, and therefrom the crack length. CCL relations are therefore developed for displacements measured at the crack mouth as well as at knife-edges of various dimensions.

SIMULATION AND ANALYSIS

The geometry and loading configuration of the 4PB specimen which is commonly used is shown in Fig. 1. As shown in the figure, the ratio of the major span to the minor span is 2, with the major span taken equal to $4W$. The finite element method (FEM) was used to simulate loading in such a specimen. Since the 4PB specimen is symmetric about its mid-plane, a two-dimensional half-model of the specimen was discretized with six-noded isoparametric triangular elements. The crack tip region was enmeshed finely with similar elements in which the mid-side nodes are moved to quarter-point positions, so that the strain singularity in this region can be accommodated. The finite element mesh was optimized by comparing the stress intensity factor K calculated from displacements local to the crack tip with that obtained from the K-expression given by Brown and Srawley [6] for the 4PB specimen. Figure 2 shows the mesh generated for $a/W = 0.5$. In order to obtain knife-edge compliance data, knife-edges characterized by z/W values of 0.075, 0.1, 0.125 and 0.15, where z is the knife-edge thickness (see Fig. 1), were modelled by two extra elements.

Finite element models containing cracks of lengths 0.25 W to 0.8 W in increments of 0.05 W were subjected to unit thickness loads of 600–3000 kN in steps of 300 kN. Using a linear elastic

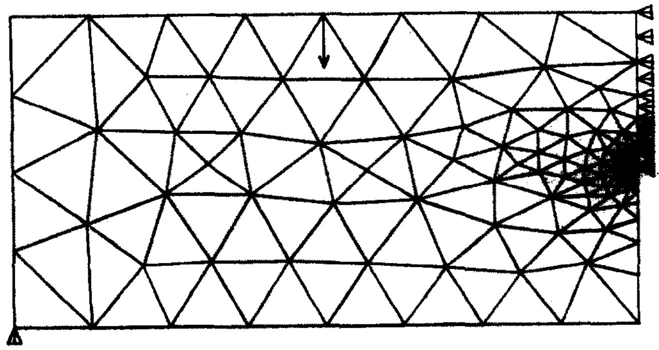

Fig. 2. The finite element mesh generated for the 4PB specimen with $a/W = 0.5$.

Table 1. Coefficients of CCL relations

Cases	A_0	A_1	A_2	A_3	A_4	A_5
Crack mouth	1.01562	−3.06851	4.53990	−23.32393	71.16366	−69.78915
$z/W = 0.075$	1.02592	−3.45135	7.46375	−38.39358	108.86220	−104.69283
$z/W = 0.1$	1.02284	−3.40410	6.65723	−34.57981	99.98037	−95.71740
$z/W = 0.125$	1.01889	−3.33493	5.67992	−30.33240	91.49075	−88.78606
$z/W = 0.15$	1.01893	−3.37365	5.79323	−31.17801	93.94080	−90.30191

analysis, and taking $E = 200$ GPa, crack opening displacement at the crack mouth and at the knife-edge was recorded at each load step. The compliance V/P of the specimen was obtained from the slope of the load–displacement data, and u, as given by eq. (3), was calculated for each crack length. Fitting a polynomial by the least square technique to the $(u, a/W)$ data for the crack mouth and each case of knife-edge considered, the corresponding CCL relation was obtained. A fifth-degree polynomial has been used to represent CCL relations as the fitting error has been found to be sufficiently small and substantial improvements have not been found unless the order of the polynomial is increased considerably. Also, CCL relations for other specimen geometries have cutomarily been represented by fifth-degree polynomials [1–5].

RESULTS AND DISCUSSION

The crack mouth and knife-edge CCL relations obtained for the 4PB specimen can be written as

$$\frac{a}{W} = A_0 + A_1 u + A_2 u^2 + A_3 u^3 + A_4 u^4 + A_5 u^5, \tag{4}$$

where u is defined as in eq. (3). The coefficients of eq. (4) are listed in Table 1 for the various crack mouth and knife-edge cases considered. Graphical representations of the various CCL relations are presented in Fig. 3. The errors, emanating from curve-fitting inaccuracies, between a/W as originally modelled in the FEM simulation and that obtained by the CCL relations developed are shown in Fig. 4. It can be seen that the errors are within 0.04%, and in comparison to the experimental precision of the compliance technique [7], they can be disregarded.

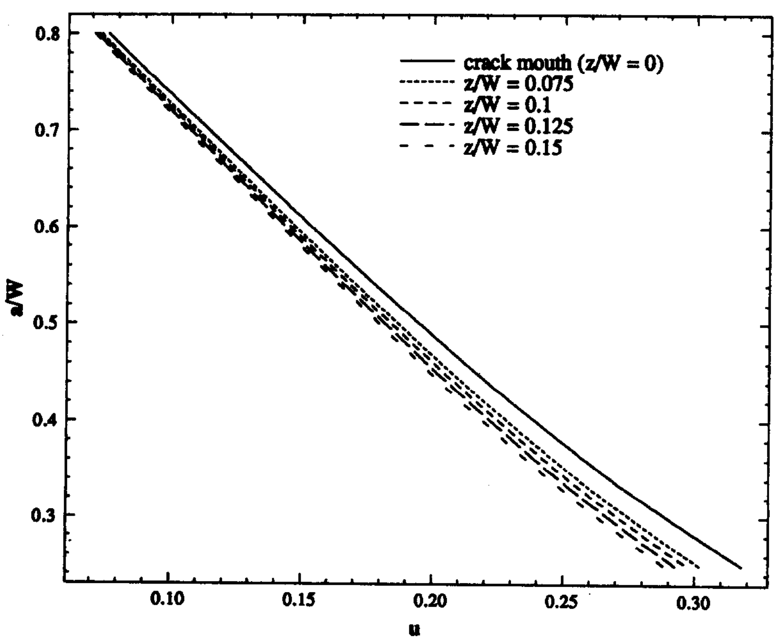

Fig. 3. Graphical representations of the various CCL relations developed.

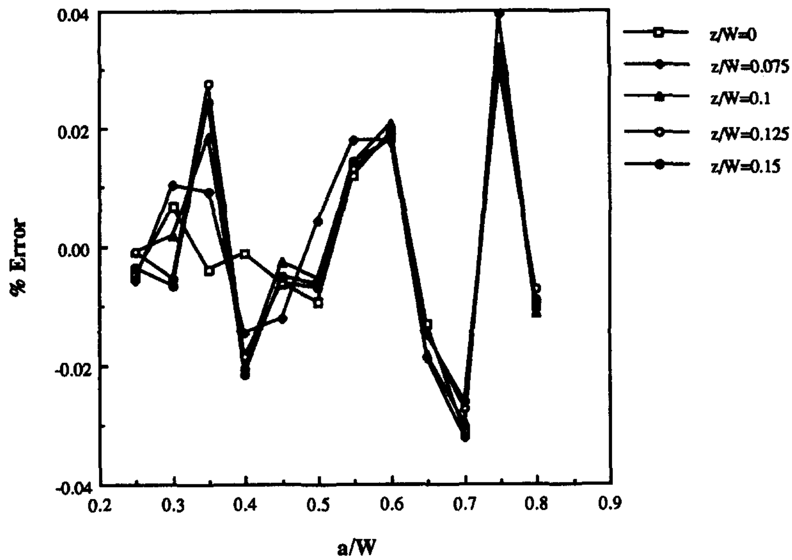

Fig. 4. Fitting error for the various CCL relations developed.

The 4PB configuration used in this investigation (Fig. 1) can be approximated to the limiting case of a pure bend situation [6] as the minor span is $2W$ and the major span is $4W$. In pure bend specimens containing an edge crack, the displacement at the crack mouth is given by

$$V = 24\frac{Ma}{EBW^2}f\left(\frac{a}{W}\right), \tag{5}$$

where M is the bending moment. The function f is reported by Tada et al. [8] to be

$$f\left(\frac{a}{W}\right) = 0.8 - 1.7\left(\frac{a}{W}\right) + 2.4\left(\frac{a}{W}\right)^2 + \frac{0.66}{\left(1-\frac{a}{W}\right)^2} \tag{6}$$

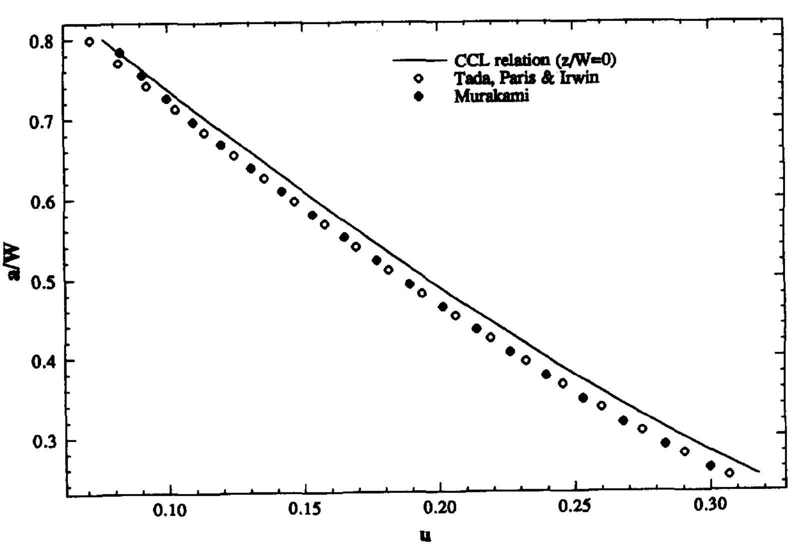

Fig. 5. Data obtained from crack mouth displacement relations given for the pure bend specimen by Tada et al. [8] and by Murakami [9], plotted along with the crack mouth CCL relation that has been developed.

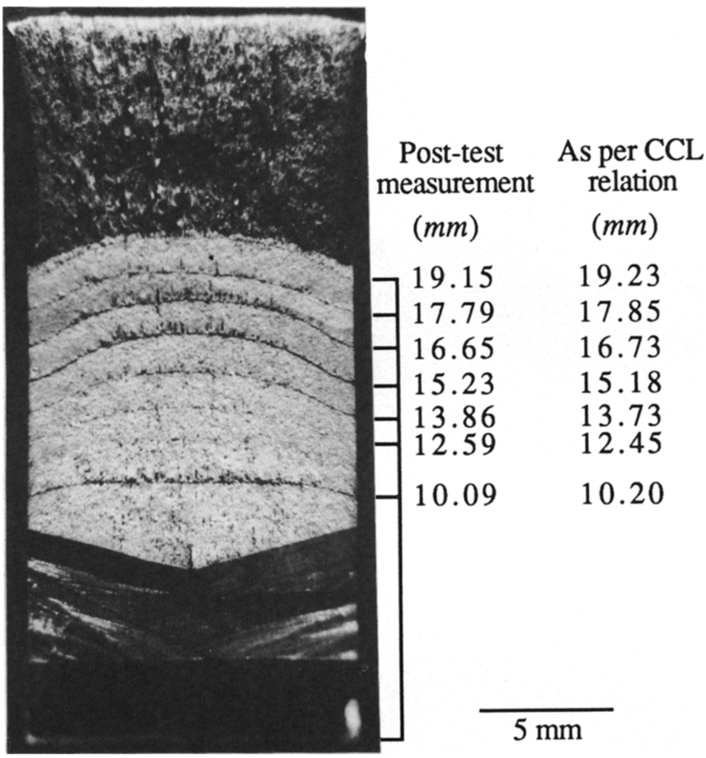

Fig. 6. A beachmarked fatigue fracture produced under 4PB loading. The crack length at each beachmark has been obtained by optical measurements (seven-point average), and compared with that obtained by the crack mouth CCL relation developed.

and by Murakami [9] as

$$f\left(\frac{a}{W}\right) = 1.458 - 0.304\left(\frac{a}{W}\right) - 0.924\left(\frac{a}{W}\right)^2 + 48.34\left(\frac{a}{W}\right)^3 - 123.5\left(\frac{a}{W}\right)^4 + 120.5\left(\frac{a}{W}\right)^5. \quad (7)$$

For the 4PB case, as $M = PS/2$, where S is the moment span which in the present configuration is equal to W, eq. (5) can be restated as

$$\frac{EBV}{P} = 12\frac{a}{W}f\left(\frac{a}{W}\right). \quad (8)$$

Using eq. (8), u was calculated as per eq. (3) for various values of a/W. The $(u, a/W)$ data thus obtained for the cases represented by eqs (6) and (7) are plotted in Fig. 5, together with the crack mouth CCL relation given by eq. (4). It can be seen that the correlation between the developed CCL relation and compliance data obtained analytically is good.

In order to experimentally validate the CCL relations developed, fatigue tests were conducted on 4PB specimens of nominal dimensions $W = 30$ mm, $B = 15$ mm and moment span $S = 30$ mm. Crack lengths were monitored using a computer interfaced to the servohydraulic testing machine through a program in which the crack mouth CCL relation was implemented. Crack lengths obtained by this method were checked against that measured optically on the specimen flanks using a travelling microscope. Taking into consideration that crack lengths are underestimated when measuring the visible crack on the specimen surface due to the slight bowing invariably associated with the crack front, the agreement was generally found to be good.

To further illustrate the performance of the CCL relations developed, a 4PB specimen was beachmarked using sequential blocks of high and low load amplitudes. The crack lengths, as obtained by using the crack mouth CCL relation, were noted at the end of each block of loading (i.e. at each beachmark). Subsequent to the test, the average crack length at each beachmark was measured optically by the seven-point averaging method. The crack lengths obtained by both the methods are shown in Fig. 6 in relation to the fracture surface. It can be seen that they correlate very well.

CONCLUSIONS

CCL relations, useful for automated fatigue testing, have been developed for the 4PB specimen geometry in a form prevalent for other test specimen geometries. The relations reported here are for compliance measured at the crack mouth and at knife-edges of various thicknesses. The crack mouth CCL relation has been found to exhibit good correlation with compliance data obtained from crack mouth displacement relations for pure bend specimens. It has also been validated experimentally by direct measurement of crack lengths.

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