ENG (NML): 2-854: Mechanical Behaviour of Materials (MBM)

In the last lecture we saw that the transformation equations for stress components can be written as

$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \sigma_{xy}\sin 2\theta \qquad \text{...(1)}$$

$$\sigma_y' = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \sigma_{xy}\cos 2\theta \qquad \text{...(2)}$$

$$\sigma_{xy}' = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \sigma_{xy}\cos 2\theta \qquad \text{...(3)}$$

If we square and add eqns (1) and (3) we get,

$$\left(\sigma_x' - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \sigma'^2_{xy} = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2\cos^2 2\theta + \sigma^2_{xy}\sin^2 2\theta + \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 \sin^2 2\theta + \sigma^2_{xy}\cos^2 2\theta$$
$$+2\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta \sin 2\theta - 2\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta\sin 2\theta$$

Upon simplifying terms

$$\left(\sigma_x' - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \sigma'^2_{xy} = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \sigma^2_{xy} \text{ ....(A)}$$

If you recall, the term on the RHS is nothing but R^2 (square of radius that we saw in the previous lecture and the term ($\sigma_x+\sigma_y/2$) is nothing but the average stress σ_{avg}.

$$(\sigma_x' - \sigma_{avg})^2 + \sigma'^2_{xy} = R^2$$

In this equation, σ_{avg} and R^2 are known. They are the original values of the stress components before transformation. σ_x' and σ_{xy}' are the variables whose value depend upon the transformation angle θ.

In generic mathematical term, the above equation can therefore be written as

$$(x-A)^2 + y^2 = R^2 \qquad \text{...(B)}$$

Because, it is customary to denote the variables by X and Y and the constants (or known values) by A, B... in general mathematics.

From the above equation (B), you may note that it is the functional form of a circle with a radius R.

In other words, the transformation of stresses can be represented in the form of a circle, known as Mohr's circle, first proposed by Otto Mohr. Note that the Mohr's circle is an elegant way of representing stress transformations in a graphical form and is a handy tool for design engineers for stress computations.

We will now see how to construct the Mohr's Circle.

<u>Construction of Mohr's Circle</u>

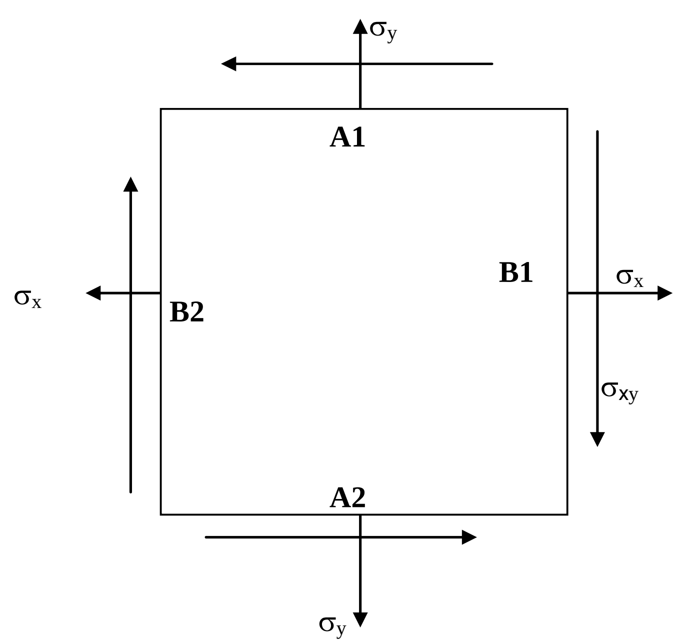
The primary requisite for the construction of Mohr's circle is the sign convention. In principle we follow the sign convention for normal stresses as given below.

Tension: Positive Compression: Negative

For shear stress, we have throughout the lecture followed the procedure as given below.

Look at the shear component. If it tends to shear the surface in clock-wise direction, we considered it positive. If it tends to shear the surface in counter clock wise direction, we took it as negative. This is called the left hand rule. It is not necessary that you have to follow only this convention. You may choose to follow the right hand rule where counter clock rotation is positive and clock wise rotation is negative. But then you have to be consistent in your approach. Since we have been using 'left hand rule' throughout this course, let us adopt this sign convention.

Let us consider a rigid body with forces acting on it as shown below for which we are going to construct the Mohr's circle.



Let us label the surfaces as A1, B1, A2 and B2. Now we have to find the co-ordinates of the plane A1 and B1.

On A1 plane, you have

Normal stress in tension, therefore POSITIVE

Shear Stress in counter clock direction, therefore POSITIVE

Co-ordinates of A1 = (σ_x, τ_{xy})

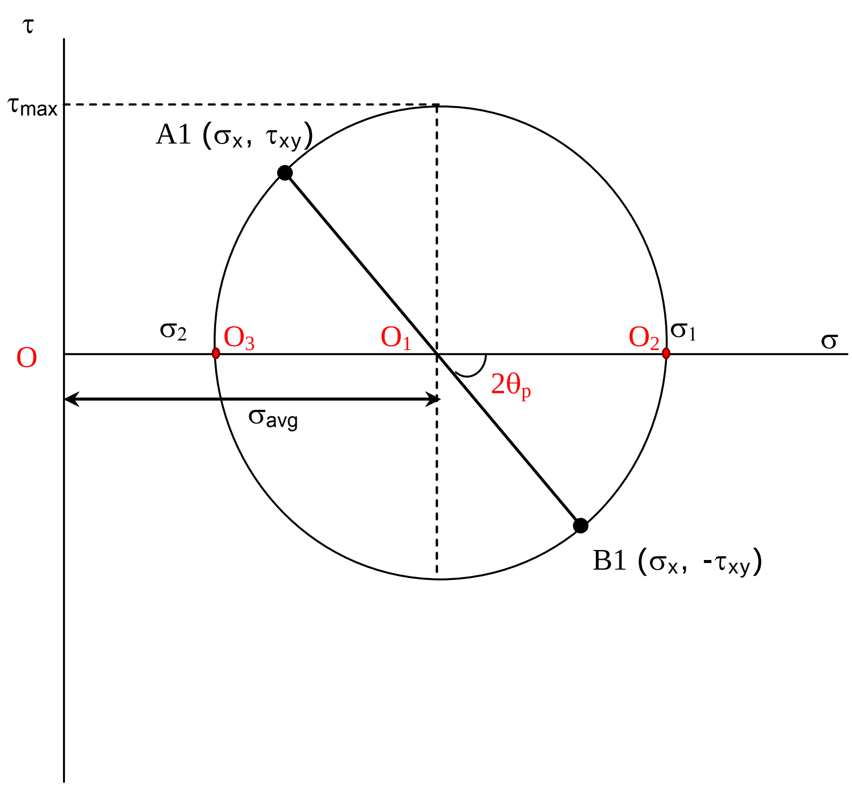
On B1 plane, you have:

Normal stress in tension, therefore POSITIVE

Shear stress clockwise direction, therefore, Negative

Co-ordinates of B1= (σ_x, -τ_{xy})

Now you draw a horizontal axis with normal stress and vertical axis with shear stress as shown in Fig below.



Plot the A1 and B1 points

Draw a circle with A1B1 as diameter.

Suppose if you had rotated the plane through Principal angle (θ_p) [note that in Mohrs circle the angle will become twice and hence θ_p], then we can find out the following

Principal Stress: It is the maximum and minimum value of the normal stress on a plane where shear stress is zero. Obviousely, this plane is the horizontal line and the max value of principal stress is σ_1 and minimum value of principal stress is σ_2 as marked in the diagram. The center of the circle is the σ_{avg} $=\left(\frac{\sigma_x+\sigma_y}{2}\right)$

We can derive these values from the following procedure

$$\sigma_1 = OO_2$$

$$= OO_1 + O_1O_2$$

$$= \left(\frac{\sigma_x+\sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

(since OO_1 is the σ_{avg} & O_1O_2 is the radius of the circle whose derivation we made in the last notes using Pythagoras theorem and repeated in eqn (A))

$$\sigma_1 = OO_3$$

$$= OO_1 + O_1O_3$$

$$= \left(\frac{\sigma_x+\sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

(since O_1O_3 is again the radius of the circle)

Max value of shear stress is nothing but the radius of the circle

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

All these valuses you can directly get from the above graph directly. The advantage of the Mohr's circle is that if you rotate the plane A1B1 to various other angles, you can directly read the values of the transformed stress components. All that you require is the original stress components and correct usage of sign convention.