Energy requirements in size reduction of solids

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INTRODUCTION :

It is no exageration to state that of the many approaches to understand different aspects of process of comminution, perhaps the most important one is the search for the sound relationship between the size and the energy used. Basic laboratory investigations of comminution have been concerned mainly with 3 phases of the problem :

1) the micro crack pattern and its progress,

2) fragment size distribution of commuted products,

3) new surface production as a relation to energy input.

In the present analysis emphasis is on the relationship between the energy utilised in comminution and new surface produced although various other criteria have also been discussed and compared with the theoretical to practical results.

Energy input and comminution efficiency :

The previous data on quartz by Piret et. al and others[1,2,3,4,5] together with author's experimental results of dry and wet grinding of quartz have been presented in Fig. 1.

From figure No. 1, it can be seen that :

1) the values given by Smekal[6] and Edsor[7] are close to each other and represent highest values showing theoretical maximum efficiency, the experimental points falling below. The theoretical values indicate the production of 395 and 370 hectares of quartz surface with the input of 1 kWh respectively.

2) the efficiency decreases from crushing to grinding steadily; the upper most point 'p' representing 25% and lower most point indicating 0.02%. This lowest point is obtained with energy input of 2250 kWh/t.

3) the applicability of three well known laws also is indicated. It is seen that the author's[10] results are in excellent aggreement with those of Svensson[9] and Murkes in the range of 1 to 35 kWh/t, sensibly following the Rittinger's[4] theory.

4) the highest efficiency in all these laboratory experiments is obtained in crushing of single particle.

Hukki and Reddy[11] illustrated by conducting experiments on grinding that the theories propounded by Kick, Rittinger and Bond have limited application and in fact in fine grinding range the relationship will be totally different as illustrated in figures 1, 2 and 3. The conclusions drawn from this study are as follows :

From the graphical representation of new surface production vs. net energy consumption, previously known and new laboratory data of quartz indicate that the highest values for new specific surface production are obtained by crushing single particle at low energy concentrations. With the increase in energy concentrations there is decrease in efficiency in comminution and decrease in production of specific surface.

Energy-particle size relationship :

The three laws of comminution are stated as under :

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1) Kick's law[12] : Kick stated that the energy required to bring analogous changes in configuration of geometrically similar bodies of equal technological character is proportional to the volumes of these bodies.

2) Rittinger's theory[4] : Von Rittinger proposed a theory stating that the energy consumed in comminution is proportional to new surface produced.

3) Bond's theory[13] : Bond's so called third theory of comminution states that the energy required is proportional to the length of crack initiating breakage.

Each of the three well known laws of comminution can be represented as a special case of general equation (1) derived by Walker, Lewis, McAdams[14] and Gilliland.

$$dE = -C dx / x^n \quad \ldots \quad (1)$$

Where E is the net energy for unit weight of the material in a particular comminution process, x is a factor indicating the fineness of material, n is an exponential factor indicating the order of the process and C is an experimental constant depending upon the material, units chosen — etc.

If in the equation (1), the exponent n is replaced by numerical figures 2, 1 and $1\frac{1}{2}$ the integrated form of the general equation leads to the well known fundamental theories of Rittinger, Kick and Bond respectively.

The total net energies (E_t) (kWh/t) from infinite feed size to a product of size x are given as follows :

Rittinger's theory $E_t = C\ 1/x$ ... (2)

Kick's law $E_t = -C\ \ell n\ x$ ... (3)

Bond's theory $E_t = 2C\ 1/\sqrt{x}$ ... (4)

For all practical purposes measurement of net energy to bring about size reduction of known feed size to desired product size is of importance. The corresponding net energies (E) (kWh/t) required for feed x_1 to product x_2 are :

Rittinger's theory $E = C\ (1/x_2 - 1/x_1)$ ... (5)

Kick's law $E = -\ \ell\ n\ (x_2 / x_1)$ ... (6)

Bond $E = 2\ C\ (1/\sqrt{x_2} - 1/\sqrt{x_1}\ )$. (7)

The three relationships indicated by equations (5), (6) and (7) if plotted on a logarthmic paper with particle size on abscissa and net energy consumption on ordinate, each of the said relationships will be represented by a straight line with a definite slope 'm'. The numerical values of these slopes will be —1.0, 0 and —0.5 respectively.

Hukki[3] has proposed a solomonic settlement between the three laws of comminution, indicating possible applicability of each law in a different size range. Later Hukki and Reddy[11] have shown, basing upon experimental data on quartz that :

— the range of crushing is the range of basic law of Kick. The slope of cumulative curve varies from $m = 0$ to $m \approx 0.1$.

— the range of coarse grinding is the range of Bond's theory, ($m = -0.5$) and the range of fine grinding is the range of Rittinger's theory ($m = -1.0$).

— the range of fine grinding from $m = -1$ to -3.

— the range of approaching grinding limit $m \approx -3$ to $m = -\infty$

In figure 3 the relationships between net energy input and particle size (mean value) are shown. The particle is calculated by equation (1) from the surface area values given in figure 2. It can be seen that curve A tends to become vertical in very fine size range i. e., $m = -\infty$ indicating that none of the present existing laws are applicable in fine size range.

It may be said that extremely fine particles do exist even when bulk of the material is in coarse form. The grind limit is conveniently defined here as the limiting particle size after which further size reduction is almost impossible under a given set of operating conditions. The mean particle size is a rational measure of the whole lot and is not necessarily the finest size. It may be recalled that Kondo and Gaudin[13] have found that finest particles in a ground product could fall in the size range of 0.03 μ.

Probability of breakage :

In crushing operations overall probability of particle being crushed in comminution is represented by figure 4 which indicates that under the conditions where all particles are broken the probability of breakage is 1.0, if one half of the mass of the particle is broken the probability factor is 0.5 and if none of the particles are broken the probability factor is zero. In crushing large material it is expected that the probability factor is high. In fine grinding it is low. In submicron grinding by conventional means it approaches zero. The nature of phenomena encountered in grinding powders is demonstrated in the Table 1.

The data given in the Table 1 gives a panoramic presentation of the theoretical subdivision of a 1 cm cube in decadic steps down to the cubical unit crystals. The following observations can be made :

— The total number of cubes formed increases in 1000—fold steps for each 10—fold step of size reduction.

— The total specific surface increases in 10—fold steps. The overall area on elementary 5 A^0 cubes totalling 1 cm^3 in volume = 12,000 m^2 or 1.2 hectares.

— The total edge length increases again in 1000—fold steps. The length of edges on 0.01 micron cubes totalling 1 cm^3 in volume = 1.2×10^8 km; the mean distance between earth and sun is about 1.5×10^8 km. The length of edges on elementary 5 A^0 cubes totalling 1 cm^3 in volume $=4.8\times10^{10}$ km; this corresponds to 320 times the mean distance between earth and sun.

Thus in the micron size range the coarsest particles loose their individuality, and the charge, both wet and dry behaves on the whole rather like a homogeneous, fluid mass

Probability Approach :

The phenomena that occurs in comminution with reference to the behaviour of particle size with the energy input breakage can perhaps be best explained by attributing probability approach to the process of comminution. Any solid matter is composed of unit crystals which can be assumed as the smallest basic limits showing the basic characteristics of the substance. In single particle breakage it is expected that the probability of achieving the breakage is high and in case of multiple particle assemblage the probability of breakage of each particle would be drastically reduced. The particle calculation with reference to the behaviour of quartz particle when subjected to comminution in 10 fold steps is indicated in table 1. Assuming that in the actual operation also a similar situation occurs if not exactly as in the particle calculations the operation of crushing and grinding can be explained by a simple model as under :

The model can be assumed as a typical laboratory ball mill of convenient size. Assuming that relative grinding capacity of 1,000 of the base mill (100 cm $\times$ 100 cm) the dimensions in centimetres obtained would be 26.8 cm $\times$ 26.8 cm. The total volume of this mill will be 15 dm^3 . A 30% volumetric ball mill corresponds to 4.5 dm^3 of steel balls weighing about 4.8 kg. 10 mm ball would weigh about 4.1 gms. The total number of balls in the charge will then be 5,270. In the mill a batch of material 1.5 kg can conveniently be ground. Assuming a density of 3.0 for volume of 1.5 kg material to be ground would be 500 cm^3 . However for the simplified model as proposed, now the original batch can be considered to consist of 500 pieces of 1 cm cubes. The 500 pieces of 1 cm cubes are now reduced ideally step by step to imaginary cubes of succeeding decadic size classes. As illustrated in Table 2, the theoretical number of particles and number of 10 mm balls available in the batch when ground step wise in the said laboratory mill the phenomena can be observed as follows :

a) The number of grinding balls remains unchanged.

b) The number of contact points, i.e., the points where further size reduction can take place, between the balls themselves and the balls and the mill lining remains unchanged.

c) With the increasing fineness the number of cubes to be broken increases at a tremendous rate for each 10—fold size reduction step the said number increases 1000—fold steps.

d) For each 10-fold size reduction step, the number of cubes for ball available in the batch increases in 1000 fold steps.

The probability of breakage as against the particle size is shown in figure 4. The curve obtained in this figure fairly approximates with the actual practice. However, depending on the grinding environment the range of size reduction etc. its position may be different but the trend of the curve would more or less be seen for all the materials.

In comminution, the probability of a particle being broken varies with the size of the particle from a top value of 1.0 applicable in the coarse range to extremely small values applicable in the extreme fine range. On a logarithmic paper showing particle size on the abscissa and the probability factor on the ordinate, the probability is represented by a curve characterized by the following features :

— Starting from the coarsest size treated, the curve has an upper horizontal straight line section with slope $m = 0$ extending substantially over the whole range covered by crushing; the probability factor = 1.0.

— a curved middle section with its positive slope increasing gradually from $m = 0$ to $m = 3.0$ with decreasing particle size, the probability factor decreases from 1.0 at a rapid rate. This section corresponds to the common range of industrial grinding.

— a lower linear section having steep slope of $m = 3.0$; the probability factor decreases in 1000—fold steps for each 10—fold size reduction step. This section corresponds to the very finest size classes.

The probability concept can also be explained in terms of the efficiency of the application of mechanical energy in comminution. It is easy to visualise that the said efficiency of the positively acting crusher is high. The net power (the total power minus the idling power) is used for reduction of particle size. In a rod mill the net power (total power minus the idling power of an empty mill) is used for tumbling the rod load and the material change. It can be understood that while the probability of a rod to hit relatively coarse particles is high, a rod may also strike another rod or the mill lining without a properly directed blow on the material to be ground. The overall efficiency of the application of mechanical energy is now lower than that of a crusher. Toward fine grinding in ball mills, the said efficiency decreases at a rapid rate with decreasing particle size and with vastly increasing particle numbers.

Practical Approach :

In conventional industrial coarse crushing about 0.5 kWh of net energy is needed to crush one ton of ore or rock from an initial size of 1 m down to 10 cm. Assuming this basic 0.5 kWh/t as net energy consumption figure needed in each decadic size reduction step, in order to get the actual net energy consumption in each step, the basic figure should be corrected regarding the relative probability of breakage in each reduction step. The correction factor is the inverse value of probability factor. An example illustrating the evaluation of energy consumption in successive decadic steps of size reduction is given in table 3. The respective probability factors are obtained from the curve shown in figure 4.

Weight Distribution Vs. Fineness :

Surface area relationship is the weight distribution — fineness in ground products. On log-log paper the well known Gates—Gaudin—Schuhmann relationship plotted as straightline. As early as 1909 a similar straightline relationship was presented by Richards. Rosin, Rammler and Sperling formulated another straightline relationship plotted on a log-log vs. log paper. These relationships fortunately have been widely accepted although these pre-opinions have not discussed the limitations relative to the particle size range.

However, one of the practical advantages in support of these straightline relationships is that they provide a basis for mathematical estimates of the specific surface area of a ground product. These estimates of specific surface area based on the size distribution of particles should be accepted within reasonable limits to agree with the surface area actually measured. However, they usually do not. One of the attributes for this discrepancy is that the mathematical estimates do not incorporate the values for samples of lower particle size limit. In fact it is difficult to estimate because the basic information does not exist and as such no reasonable recommendation is proposed for establishing its value and consequent connections to be made to reconcile between the estimated and actual surface area of any powder.

Subsequent studies have indicated that contrary to the conventional method of plotting as proposed by Gaudin—Gates—Schuhmann are limited to certain size range only and not applicable for the whole range of comminution starting from coarse crushing to fine grinding. In fact the data covering the full range of comminution when plotted as proposed by Gates—Gaudin—Schuhmann reveals that it is no more straightline with a definite slope but in fact smooth curve. In the coarse range however the curve is normally straightline with a definite slope but slightly taking at 100% passing mesh size. A typical example is presented in figure 5 wherein the size distribution of ball mill feed and products have different times of grind are illustrated.

The range approaching grind limit of any particular mineral is expected that the slope vary from -3 to $-\infty$. Generally this range is of purely theoretical importance and can not have any practical applications.

Closed Circuit Grinding :

The general response of solids to breakage process resulting in a parabolic shape curve when plotted on log-log paper such as shown in figure 3 is well illustrated for a typical case of size reduction of quartz. It is expected that similar pattern of curves can be obtained for various other minerals also. So is the case for comminuted products in close circuit. The relationship between energy input and new surface produced is illustrated by curve B in figure 2 by Hukki. However, in case of closed circuit, the corresponding energy requirements for a given particle size will be lesser throughout the range of comminution. The following conclusions can be drawn from the above analysis :

— The crushing range is covered by the basic law of Kick, where the slope of the cumulative curve varies between 0 to —0.1.

— In the case of grinding i.e., the size reduction carried out in conventional product and ball mill operations, Bond's theory is reasonably applicable. The slope of the curve is about 0.5.

— In fine grinding range such as grinding of cement etc. Rittinger's law is applicable. The slope of the curve would be approximately—1,

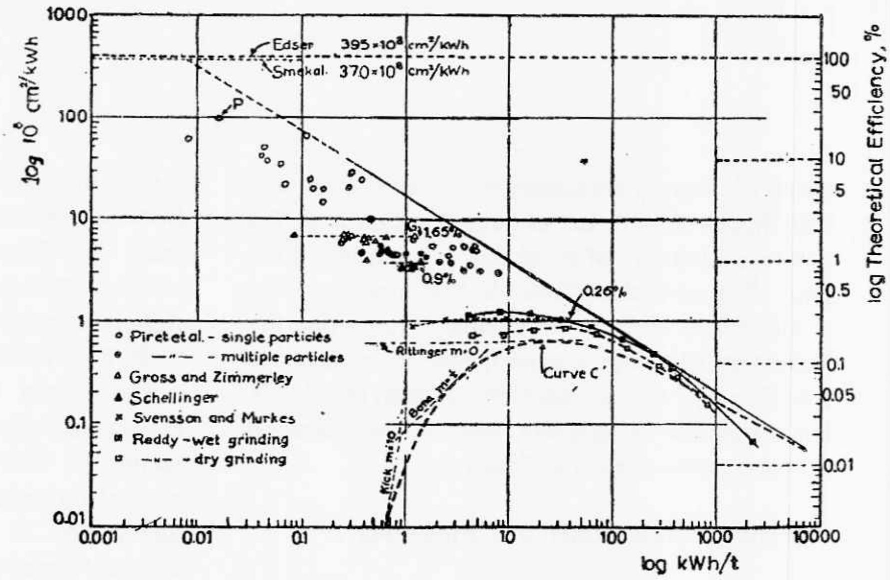

Fig. 1 : Compiled logarithmic presentation of specific surface production against net energy input.

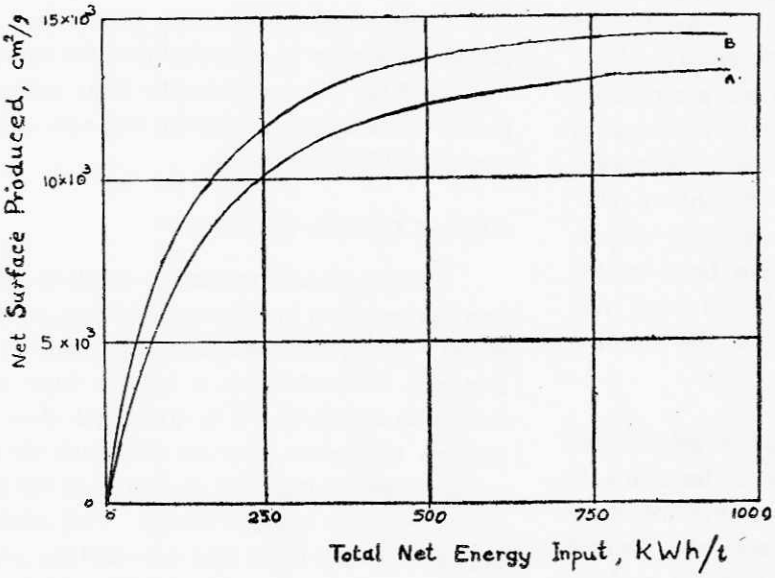

Fig. 2 : New surface area-Net energy relationship. Curve A—open circut grinding, Curve B—closed circuit grinding. (Quartz).

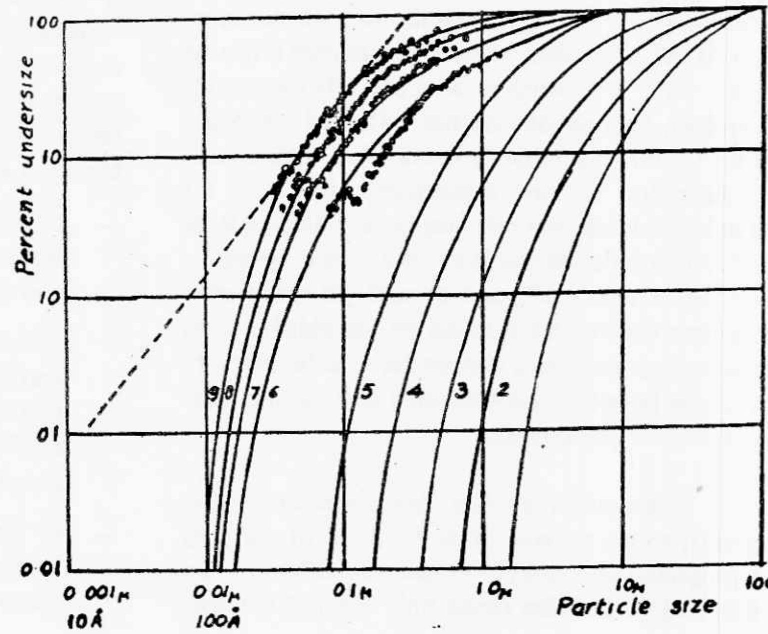

Fig. 5 : Size distribution Vs. fineness.

— In case of micro fine grinding range the slope of the curve would be approximately —1 to —3.

With regards to energy consumption in closed circuit grinding, it would be appropriate to correlate it with fineness of classification, unlike in open circuit grinding where the whole lot of ground particulate material is taken into account.

It is well established that while energy in certain closed circuit sizing steps (such as screening of crushed products, rake and spiral classified ground products) may seem of minor importance, this energy factor rapidly increases in closed circuit processes related to production of finer products.

Increasing fineness of the classified product requires increasing energy consumption for classification. This fundamental relation is valid for both hydraulic and pneumatic classification.

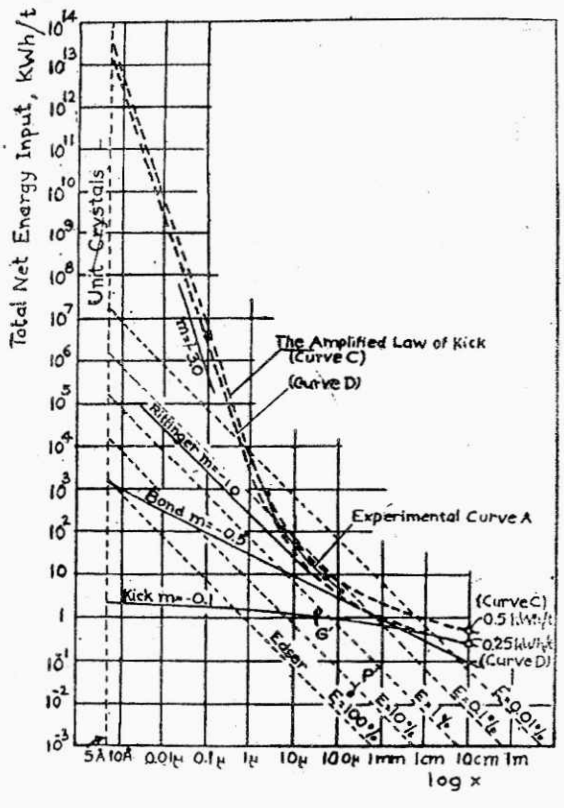

Fig. 3 : Net energy input as against particle size.

Conclusions :

A panoramic view of the energy-size relationship is presented and discussed. The validity of the three laws of comminution has been discussed.

The energy-particle size relationship in fine particle size range is explained by probability approach. It has been concluded that Kick's law is applicable to coarse crushing range, Bond's law in coarse grinding, Rittinger's theory in fine grinding. For extremely fine grinding range approaching grind limit new proposals have now been made.

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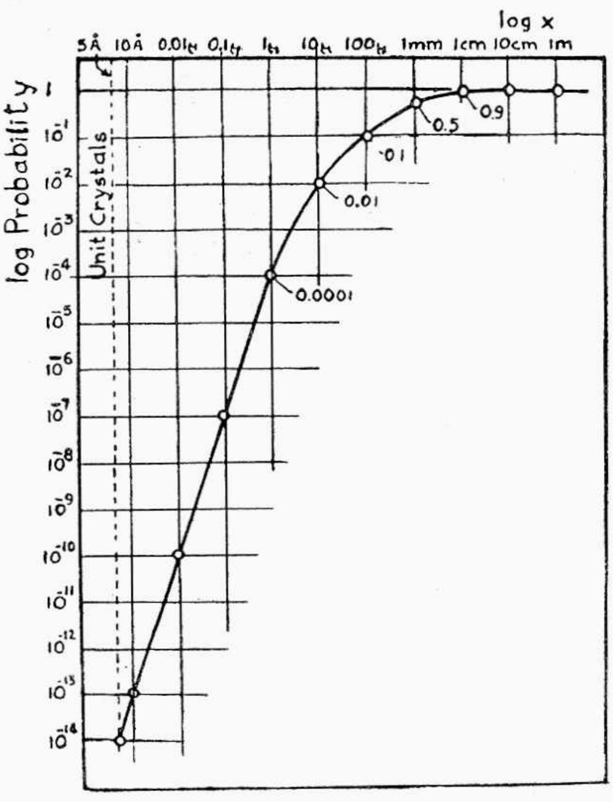

Fig. 4 : Probability of breakage as against particle size.

TABLE—1 : Characteristics of cubical products theoritically obtained by size reduction in decadic steps

1. Size of cubical particle	1 cm	1 mm	100 μ	10 μ	1 μ	0.1 μ	0.01 μ	0.001 μ	5 Å
2. Total number of cubes	1	1×10^{3}	1×10^{6}	1×10^{9}	1×10^{12}	1×10^{15}	1×10^{18}	1×10^{21}	8×10^{21}
3. Total specific surface / cm^3	$6cm^2$	$60cm^2$	$6dm^2$	$60dm^2$	$6m^2$	$60m^2$	$600m^2$	$6000m^2$	$12000m^{21}$
4. Total length of edges	12 cm	12m	1.2 km	1.2×10^{2} km	1.2×10^{4} km	1.2×10^{6} km	1.2×10^{8} km x)	1.2×10^{10} km	3×10^{10} km xx)
5. Total number of corners	8	8×10^{3}	8×10^{6}	8×10^{9}	8×10^{12}	8×10^{15}	8×10^{18}	8×10^{21}	6.4×10^{22}
6. Length of edges per cm^2	2cm	2dm	2m	20m	200m	2km	20km	200km	250km
7. Number of corners per cm^2	1.3	1.3×10^{2}	1.3×10^{4}	1.3×10^{6}	1.3×10^{8}	1.3×10^{10}	1.3×10^{12}	1.3×10^{14}	5.3×10^{14}
8. Total surface bonds	2.4×10^{15}	2.6×10^{16}	2.7×10^{17}	2.4×10^{18}	2.4×10^{19}	2.4×10^{20}	2.4×10^{21}	2.4×10^{22}	4.8×10^{22}
9. Total edge bonds	$2 \times 2.4 \times 10^{8}$	$2 \times 2.4 \times 10^{10}$	$2 \times 2.4 \times 10^{12}$	$2 \times 2.4 \times 10^{14}$	$2 \times 2.4 \times 10^{16}$	$2 \times 2.4 \times 10^{18}$	$2 \times 2.4 \times 10^{20}$	—	—
10. Total corner bonds	$3 \times 8 \times 10$	$3 \times 8 \times 10^{3}$	$3 \times 8 \times 10^{6}$	$3 \times 8 \times 10^{9}$	$3 \times 8 \times 10^{12}$	$3 \times 8 \times 10^{15}$	$3 \times 8 \times 10^{18}$	$3 \times 8 \times 10^{21}$	—
11. Percent distn. of bonds	100:0:0	100:0:0	100:0:0	100:0:0	99.8:0.2:0	98.1:1.9:0	80:19:1	0:0:100	—

Table 2 : Theoritical distribution of total number of cubes per ball in various size classes

Size class	Theoritical number of cubes	Number of Φ 10 mm balls	Number of cubes per ball
1 cm	500	5000	0.1
1 mm	500×10^3	5000	100
100 μ	500×10^6	5000	1×10^5
10 μ	500×10^{12}	5000	1×10^{11}
1.0 μ	500×10^{15}	5000	1×10^{14}
100 Å	500×10^{18}	5000	1×10^{17}
10 Å	500×10^{21}	5000	1×10^{20}

Table 3 : Evaluation of energy required for decadic steps of size reduction

Reduction step	Basic net energy per step, kWh/t	Probability factor	Total net energy per kWh/t step	Cumulative total net energy kWh/t
From 1 m to 10 cm	0.5	1.0	0.50	0.50
From 10 cm to 1 cm	0.5	0.9	0.55	1.05
From 1 cm to 1 mm	0.5	0.5	1.00	2.05
From 1 mm to 100 μ	0.5	0.1	5.00	7.00
From 100 μ to 10 μ	0.5	0.01	50.00	57.00
From 10 μ to 1 μ	0.5	1×10^{-4}	5000.00	5057.00
From 1 μ to 0.1 μ	0.5	1×10^{-7}	5000000.00	5005000.00

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