Sizing a mill — a statistical approach

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Introduction :

In an ore-dressing plant, the energy consumption (kwh/t) in grinding constitutes about 70% of the total processing cost. Thus the energy consumption of a grinding mill becomes an important factor in cost economics of the process. Of the several factors affecting the energy consumption of grinding mill, the size (dia) of the mill is more important. It is also seen that maximum energy consumption(1) is attributed to the ball load as it constitutes 45-50% of the mill volume.

In this paper, a log-log relationship is obtained between power consumption (kwh/t) and i) ball load and ii) mill-diameter. A knowledge of Bond's work index and further the total power for the mill throughput are known a priori, this power consumption is then utilized to find ball load and/or mill size. The calculation is for a closed circuit wet grinding in O.F discharge ball mill and a simple rod mill. For grate discharge ball mill the appropriate correction factors are utilized(2).

Description :

Several complex empirical equations are developed(2) relating to mill dia, speed, ball/rod load (% mill volume), ball size. But speed varies in a narrow range viz. 75 to 80% of N_c, for ball mill and 65 to 75% of N_c for rod mill. N_c is the critical rpm of the mill. Maximum energy consumption is found to be due to media load. So for a fixed media load (% of mill volume) and a constant speed, power consumed is related to mill diameter. A log-log plot of the data(2) shows a straight line relationship.

Regression equations are determined for both the mills. The correlation co-efficient and chi-square (K^2)- test confirms the accuracy of equation.

TABLE—1

The values of n and a in $y=a\,x^n$ for mills.

Mill	% mill volume filling		
Ball mill	40	45	35
n	3.1597	3.1721	3.1379
a	12.2761	12.5846	11.90924
Rod mill			
n	3.1296	3.15232	3.08302
a	15.40636	15.8016	14.928

Application :

For an ore the work index is determined by Bond's method. With the knowledge of mill throughput and with the help of several correction factors—dry, open circuit, high/low reduction ratio, the mill power (kw at pinion shaft) is calculated. From the equations developed in this paper, the mill size is calculated.

EXAMPLE

Size an O'F ballmill with 40% ball load with the use of data :

Mill throughput (tph)	500.0
Feed size (μ m)	1200
Product size (μ m)	175
Work index, W_i	11.7

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Table — 2 : POWER REQUIREMENT IN A RODMILL

Total power at pinion shaft, w (kw) at % mill volume filling.

Dia D(m)	log D (x)	W 40%	log W (y_1)	W (45%)	log W (y_2)	W 35%	log W (y_3)	xy_1	xy_2	xy_3	x^2
.76	−.1192	5.96	0.7752	5.96	0.7752	5.22	0.7177	−.0924	−.0924	−.0855	0.01421
1.07	.0294	18.61	1.2697	19.4	1.2878	17.2	1.2355	0.0373	0.0379	0.0363	8.6436×10^{-4}
1.37	.1367	45.5	1.6580	47.7	1.6785	42.5	1.6284	0.2266	0.2294	0.2226	0.01869
1.98	.2967	145.0	2.1614	152.0	2.1818	135.0	2.1303	0.6413	0.6473	0.6321	0.0880
2.44	.3874	254.0	2.4048	268.0	2.4281	238.0	2.3766	0.9316	0.9406	0.9207	0.1501
2.55	.4065	275.8	2.4403	282.0	2.4609	257.0	2.4100	0.9920	1.0030	0.9797	0.1652
2.85	.4548	405.0	2.6074	426.0	2.6294	378.0	2.5775	1.1858	1.1958	1.1723	0.2068
3.00	.4771	486.0	2.6866	512.0	2.7092	454.0	2.6570	1.2818	1.2925	1.2676	0.2276
3.31	.5198	655.0	2.8162	689.0	2.8382	610.0	2.7853	1.4639	1.4753	1.4478	0.2702
3.46	.5391	725.0	2.8603	764.0	2.8831	676.0	2.8300	1.5420	1.5543	1.5256	0.2906
3.76	.5752	1010.0	3.0043	1061.0	3.0257	944.0	2.9750	1.7281	1.7404	1.7112	0.3308
3.92	.5933	1109.0	3.0449	1169.0	3.0678	1031.0	3.0132	1.8065	1.8201	1.7877	0.3520
4.22	.6253	1371.0	3.1370	1441.0	3.1587	1280.0	3.1072	1.9616	1.9751	1.9430	0.3920
4.37	.6405	1480.0	3.1730	1560.0	3.1931	1381.0	3.1402	2.0306	2.0452	2.0113	0.4102
N=14 Sum	5.5626		34.0391		34.3175		33.5839	15.7368	15.8646	15.5724	2.9173
Mean	.39733		2.431364		2.45125		2.399				

S_x or $\delta_{n-1}|_x = .2331$

S_{y_2} or $\delta_{n-1}|_{y_2} = .7353$

S_{y_1} or $\delta_{n-1}|_{y_1} = .7309$

S_{y_3} or $\delta_{n-1}|_{y_3} = .7359$

$$W = 10\,W_i\left(\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{F}}\right) = 10 \times 11.7\left(\frac{1}{\sqrt{175}} - \frac{1}{\sqrt{1200}}\right) = 5.47 \text{ kwh/t}$$

So total power consumption = W × throughput = 5.47 × 500 = 2735 kw

$y = a\,x^n$, y = kw, x = dia (m) inside liner, $n = 3.1296$ and $a = 12.2761$

Substituting in equation : $2735 = 12.2761\,(X)^{3.1597}$

So $X = 5.23$ (m)

Use a mill 5.53 × 5.53 inside liner or a 5.73 × 5.73 outside liner ballmill.

RODMILL REGRESSION EQUATION :

$y = a\,x^n$ or $\log y = \log a + n \log x$. Here $y = w$, $x = n$

or $Y = a_0 + a_1 X$, $Y = \log y$, $a_0 = \log a$

$$a_1 = \frac{\Sigma_{xy}}{S_{xx}} = \frac{N\,\Sigma XY - \Sigma X\,\Sigma Y}{N\,\Sigma X^2 - (\Sigma X)^2} \quad \text{and } \overline{Y} = a_0 + a_1\overline{X} \text{ or } a_0 = -a_1\overline{X} + \overline{Y}$$

$$\underset{(1)}{a_1} = \frac{(14 \times 15.73675 - 5.5626 \times 34.0364)}{(14 \times 2.9173311 - (5.5626)^2)} = \frac{30.9836}{9.9001} = 3.1296$$

same for all a_1's

$$\underset{(2)}{a_1} = \frac{(14 \times 15.8644 - 5.5626 \times 34.3175)}{D^r} = \frac{31.2083}{9.9001} = 3.15232$$

$$\underset{(3)}{a_1} = \frac{(14 \times 15.5724 - 5.5626 \times 33.5839)}{D^r} = \frac{31.20}{9.9001} = 3.08302$$

$$a_0 = -a_1\overline{X} + \overline{Y} = -\begin{pmatrix}3.12960\\3.15232\\3.08302\end{pmatrix} \times (.397331) + \begin{pmatrix}2.43120\\2.45125\\2.39900\end{pmatrix} = \begin{pmatrix}1.1877\\1.1987\\1.1740\end{pmatrix}$$

$= \log a$

$$\text{So } a = \begin{pmatrix}15.406359\\15.801560\\14\,927944\end{pmatrix}$$

Thus in equation $y = a\,x^n$ the values of a and n for different % mill volume filling are :

	% mill volume filling		
	40	45	35
n	3.12960	3.15232	3.08302
a	15.40636	15.80160	14.92800

Table — 3 : POWER REQUIREMENT IN A BALLMILL

Total power at pinion shaft, w (kw) at % mill volume filling.

Dia D(m)	log D (x)	W (40%)	log W (y_1)	W (45%)	log W (y_2)	W (35%)	log W (y_3)	xy_1	xy_2	xy_3	x_2
0.76	–.1192	5.21	0.7168	5.22	0.7176	5.22	0.7176	–.0854	–.0855	–.0855	0.0142
1.37	.1367	33.6	1.5263	35.1	1.5453	31.4	1.4969	0.2086	0.2112	0.2046	0.0187
1.98	.2967	108.0	2.0334	113.0	2.0531	102.1	2.0090	0.6033	0.6092	0.5961	0.0808
2.44	.3874	199.0	2.2988	206.1	2.3141	186.5	2.2707	0.8906	0.8965	0.8797	0.1501
2.74	.4377	291.0	2.4639	303.0	2.4814	284.0	2.4533	1.0784	1.0861	1.0738	0.1916
3.05	.4843	411.0	2.6138	429.0	2.6324	387.8	2.5886	1.2659	1.2749	1.2536	0.2346
3.32	.5211	535.0	2.7283	558.0	2.7466	502.0	2.7007	1.4217	1.4212	1.4073	0.2715
3.63	.5600	710.0	2.8512	740.0	2.8692	669.0	2.8254	1.5966	1.6067	1.5822	0.3136
3.93	.5944	944.0	2.9750	986.0	2.9939	886.0	2.9474	1.7683	1.7796	1.7519	0.3533
4.24	.6274	1180.0	3.0719	1235.0	3.0917	1115.0	3.0473	1.9273	1.9397	1.9119	0.3936
4.54	.6570	1459.0	3.1640	1519.0	3.1856	1370.0	3.1367	2.0787	2.0929	2.0608	0.4317
4.85	.6857	1768.0	3.2475	1841.0	3.2650	1661.0	3.2209	2.2268	2.2388	2.2082	0.4702
5.00	.6990	2060.0	3.3139	2110.0	3.3243	1940.0	3.2878	2.3164	2.3237	2.2982	0.4886
5.30	.7243	2440.0	3.3874	2541.0	3.4050	2290.0	3.3598	2.4535	2.4662	2.4335	0.5246
N=14 Sum	6.6925		36.3922		36.6252		36.0616	19.7506	19.8712	19.5764	3.94421
Mean	0.4780352		2.599443		2.616083		2.57583				

S_x or $\delta_{n-1}|_x = 0.239392$

S_{y_2} or $\delta_{n-1}|_{y_2} = 0.7593938$

S_{y_1} or $\delta_{n-1}|_{y_1} = 0.7564909$

S_{y_3} or $\delta_{n-1}|_{x_3} = 0.7512281$

BALLMILL REGRESSION EQUATION :

$$\underset{(1)}{a_1} = \frac{(14 \times 19.75063 - 6.6925 \times 36.3922)}{[14 \times 3.94421 - (6.6925)^2]} = \frac{32.954021}{10.4294} = 3.1597$$

Denominator same for all a_1's

$$\underset{(2)}{a_1} = \frac{(14 \times 19.87124 - 6.6925 \times 36.6252)}{D^r} = \frac{33.08321}{10.4294} = 3.1721$$

$$\underset{(3)}{a_1} = \frac{(14 \times 19.57637 - 6.6925 \times 36.0616)}{D^r} = \frac{32.7269}{10.4294} = 3.1379$$

$$a_0 = \overline{Y} - a_1\,\overline{X} = \begin{pmatrix} 2.5994 \\ 2.6161 \\ 2.5758 \end{pmatrix} - 0.4780 \begin{pmatrix} 3.1597 \\ 3.1721 \\ 3.1379 \end{pmatrix} = \begin{pmatrix} 1.08906 \\ 1.09984 \\ 1.07588 \end{pmatrix} = \log a$$

$$\text{So} \quad a = \begin{pmatrix} 12.276088 \\ 12.584617 \\ 11.909230 \end{pmatrix}$$

Thus in equation $y = a\,x^n$ the values of a and n for different % mill volume filling are :

	% mill volume filling		
	40	45	35
n	3.1597	3.1721	3.1379
a	12.2761	12.58462	11.90924

CORRELATION COEFFICIENT : (r)

$$r = a_1 \frac{S_x}{S_y} = a_1 \frac{\delta_{n-1}\big|_x}{\delta_{n-1}\big|_y}, \quad \text{where } S_x,\ S_y \equiv \delta_{n-1}\big|_{x \text{ or } y} \text{ are sample —}$$

standard deviations and a_1's are from the regression equations already determined i.e.

$$y = a_0 + a_1\,x$$

$$S_{xx} = \Sigma(X - \overline{X})(X - \overline{X}) = \Sigma(X - \overline{X})^2$$

$$S_x^2 = \frac{\Sigma(x - x)^{-2}}{(n-1)} = S_{xx} / (n-1)$$

$$\text{or } (n-1) \quad S_x^2 = S_{xx}$$

TABLE — 4 : K^2 TEST. GOODNESS OF FIT

RODMILL :

Dia,D (x)	y_1 (obs) (o)	y_1 (calc) (e)	$K_1{}^2$	y_2 (obs) (o)	y_2 (calc) (e)	$K_2{}^2$	y_3 (obs) (o)	y_3 (calc) (e)	$K_3{}^2$
.76	5.96	6.5267	0.04921	5.96	6.6525	.0721	5.22	6.4054	0.2194
1.07	18.61	19.0397	0.0097	19.40	19.5582	.0013	17.20	18.3905	0.0771
1.37	45.50	41.2649	0.4347	47.70	42.6273	.6037	42.50	39.4016	0.2437
1.98	145.00	130.6602	1.5738	152.00	136.1083	1.8555	135.00	122.6384	1.2460
2.44	254.00	251.2321	0.0305	268.00	262.9528	.0969	238.00	233.5244	0.0858
2.55	275.80	288.4086	0.5512	289.00	302.1661	.5737	257.00	267.5304	0.4145
2.85	405.00	408.4910	0.0298	426.00	429.0596	.0218	378.00	376.9619	0.0029
3.00	486.00	479.6215	0.0848	512.00	504.3591	.1158	454.00	441.5460	0.3513
3.31	655.00	652.4598	0.0099	689.00	687.6465	.0027	610.00	597.9181	0.2441
3.46	725.00	749.5361	0.8031	764.00	790.7541	.9052	676.00	685.4630	0.1306
3.76	1010.00	972.3171	1.4604	1061.00	1027.7256	1.0773	944.00	885.7624	3.8290
3.92	1109.00	1107.7662	0.0014	1169.00	1172.0026	.0077	1031.00	1007.1970	0.5625
4.22	1371.00	1395.3330	0.4243	1441.00	1478.7202	.9622	1280.00	1264.3066	0.1948
4.37	1480.00	1556.5057	3.7604	1560.00	1650.8343	4.9980	1381.00	1408.0520	0.5797
Sum			9.2234			11.2937			8.1214

$K^2_{(.05,13)} = 22.36$, All values calculated above for different W's are below this critical value. Hence equations are well-fitting.

Where :

$K^2 = (o - e)^2 / e$; e = Calculated Value ; o = Observed Value

$y_1 = 15.4064 \times {}^{3.1296}$ $y_2 = 15.8016 \times {}^{3.1523}$ $y_3 = 14.928 \times {}^{3.0830}$

(w 40) (w 45) (35)

TABLE — 5 : K^2 TEST. GOODNESS OF FIT

BALLMILL :

Dia,D (x)	y_1 (obs) (o)	y_1 (calc) (e)	K_1^2	y_2 (obs) (o)	y_2 (calc) (e)	K_2^2	y_3 (obs) (o)	y_3 (calc) (e)	K_3^2
0.76	5.21	5.1578	0.4823	5.22	5.2695	0.0005	5.22	5.0337	0.00689
1.37	33.6	33.1938	0.0050	35.1	34.1611	0.0258	31.40	31.9815	0.01058
1.98	108.0	106.2757	0.0280	113.0	109.8731	0.0890	102.10	101.5757	0.0027
2.44	199.0	205.6345	0.2141	206.1	213.1470	0.2330	186.50	195.6476	0.4277
2.74	291.0	296.6340	0.1070	303.0	307.9134	0.0784	284.00	281.5150	0.0219
3.05	411.0	416.2005	0.0650	429.0	432.6011	0.0300	387.80	394.0655	0.0996
3.32	535.0	544.1271	0.1531	558.0	566.1640	0.1177	502.00	509.3922	0.1073
3.63	710.0	721.4353	0.1812	740.0	751.4843	0.1755	669.00	680.4795	0.1937
3.93	944.0	927.1768	0.3052	986.0	966.9467	0.3754	886.00	873.0285	0.1927
4.25	1180.0	1187.3559	0.0456	1235.0	1239.2320	0.0145	1115.00	1116.1065	0.0012
4.54	1459.0	1462.7152	0.0094	1519.0	1560.1281	1.0842	1370.00	1372.9654	0.0064
4.85	1768.0	1802.1818	0.6483	1841.0	1884.0024	0.9815	1661.00	1689.1688	0.4698
5 00	2060.0	1984.2479	2.8920	2110.0	2075.1182	0.5864	1940.00	1858.5832	3.5669
5.30	2440.0	2385.3651	1.2514	2541.0	2496.4081	0.7466	2290.00	2231.4609	1.5357
Sum			6.3876			4.5884			6.6425

$K^2_{(.05,13)}$ = 22.36, All values calculated above for different W's are below this critical value. Hence equations are well fitting.

Where :

$K^2 = (o - e)^2 / e$; o = Observed Value ; e = Calculated Value

$y_1 = 12.2761 \times X^{3.1597}$ (w 40)

$y_2 = 12.5846 \times X^{3.1721}$ (w 45)

$y_3 = 11.9092 \times X^{3.1379}$ (35)

RODMILL :

$$a_1 = 3.1296 \quad , r = 3.1296 \times \frac{0.2330929}{0.7309081} = 0.9980564$$
(1)

$$a_1 = 3.15232 \quad , r = 3.15232 \times \frac{0.2330929}{0.7352962} = 0.9993026$$
(2)

$$a_1 = 3.08302 \quad , r = 3.08302 \times \frac{0.2330929}{0.7359193} = 0.9765066$$
(3)

BALLMALL :

$$a_1 = 3.1597 \quad , r = 3.1597 \times \frac{0.239392}{0.7564909} = 0.9998889$$
(1)

$$a_1 = 3.1721 \quad , r = 3.1721 \times \frac{0.239392}{0.7593938} = 0.9999757$$
(2)

$$a_1 = 3.1379 \quad , r = 3.1379 \times \frac{0.239392}{0.7512281} = 0.9999468$$
(3)

Conclusions :

— Energy consumption in milling (rod and / or ball mill) constitutes about 70% of the total processing cost of an ore-dressing plant. A proper sizing of a mill is important.

– Regression equations are developed for rod and ball mills. It obviates the complicated empirical equation developed earlier (2)

— A knowledge of Bond's work index and the plant throughput is required to find the total power required for the mill which in turn establishes the size of mill through the regression equations.

– The correlation coefficients and Chi-square K^2 test are made to establish the adequacy of fit of the equations.

Acknowledgement :

The author wishes to acknowledge the encouragement given by the Director, National Metallurgical Laboratoy, Jamshedput for this work.

References :

(1) Hand Book of Mineral Dressing — Taggart A.F. pp 5—49, Power consumption, John Wilsey & Sons Inc 1927.

(2) Mineral Processing Plant Design — Mular A. L. and Bhappu R.B.

Mineral Processing Division of Society of Mining, Engineers of AIME, 2nd Edition, 1980. (American Institute of Mining, Metallurgical and Petroleum Engineers Inc)

i) pp 248, equation 3, 3a and Table V pp 250 Rod mill

ii) pp 256, equation 4, 4a and Table VII pp 258 Ball mill

APPENDIX

EMPIRICAL EQUATIONS (for power calculation) :

$$Kw_r = 1{\cdot}752\ D^{\frac{1}{3}}\ (6.3 - 5.4\ V_p)\ fN_C$$

$$Kw_b = 4.879\ D^{0.3}(3.2 - 3V_p)\ fN_C \left(1 - \frac{0.1}{2^9 - 10fN_c}\right) + S_s$$

$$S_s = 1.102\ \left(\frac{B - 12.5D}{50.8}\right)$$

Kw_r or Kw_b , the power at mill pinion shaft (Kw per tonne of rods or balls)

D, Diameter (m) inside liner of rod or ball mill

V_p , fraction of mill volume loaded with rods or balls

fN_C , fraction of critical mill speed

S_S , Kw per tonne of balls

B , ball size (mm)

WORK INDEX (W_i)

Work index W_i is kwh/t_S required to break from infinite size, F = $\propto$ to P = 100 μ m

Ballmill : $W_i = 44.5/A,\ A = P_1^{0.23}\ G_b^{0.82}\ (10/\sqrt{P} - 10/\sqrt{F})$

Rodmill : $W_i = 62/B,\ B = P_1^{0.23}\ G_r^{0.625}\ (10/\sqrt{P} - 10/\sqrt{F})$

W_i , work-index (kwh/t_S for ball – or rod mill

P_1 , mesh of grind (μ m)

P,F Product and feed size (μ m), 80% passing size

G_b , G_r net gram produced per revolution

Power (kw) at mill pinionshaft is given by :

$W = W_i\ (10/\sqrt{P} - 10/\sqrt{F})$, W (kwh / t_S). Converting to t_m by multiplying a factor 1.1 we have :

$W = 11\ W_i\ (1/\sqrt{P} - 1/\sqrt{F})$

EFFICIENCY / CORRECTION FACTORS :

Work-index (W_i is caluclated for an overflow ball rod mill 2.44 m (id), closed circuit wet grinding 250% circulating load (C. L.) with feed, F coarser than 4000 μ m and product, P not finer than 74 μm. So for an industrial practice, efficiency/correction factors are needed.

(.) ball mill, (—) rod mill, (. —) ball and rod mill

i	k_i	corrections due to
1 (.)	1.3	dry
2 (.)	1.2	open circuit
3 (.)	$(P + 10.3) / 1.145\,P$	finer grinding than 74 μm
4 (.)	$\left(1 + \frac{0.13}{R_r = 1.35}\right)$	low reduction ratio ($R_r < 6$) particulary in regrinding circuit
5 (.)	$(250 / C)^{0.1}$	C. L., Circulating load, $C > 50\ \%$
6 (.—)	$(2.44 / D)^{0.2}$	mill dia
7 (.—)	$[1 + (W_i - 7)]\ \frac{F - F_0}{F_0}\ \frac{P}{F}$ $F_0 = 16000\ (13/W_i) = x$ for rod mill $F_0 = \frac{1}{4}\,x$ for ball mill	oversize feed factor coarser than optimum feed (related to W_i)
8 (—)	$1 + (R_r - R_{ro})^2 / 150$, $R_{ro} = 8 + 5\ L/D$ No equation for $R_{ro} \pm 2 = R_r$, otherwise use it for low R_r and also for high R_r (when $W_i > 7$)	high or low R_r, reduction rate
9 (—)	(a) Rod mill only 1.4 OCC crushing feed 1.2 CCC crushing feed (b) rod and ball mill combined : 1.2 OCC crushing feed No factor for CCC crushing feed	OCC, Open circuit crushing CCC, Closed circut crushing

DISCUSSION :

S. T. Kulkarni
NML, Bombay

Question 1 : Please elaborate the change on power consumption by conventional method and by your method and how the saving is achieved ?

Author : I have not mentioned any saving in power consumption. The paper relates to the statistical analysis of a set of data and formation of generalised equation which can be used to size a mill.

M. S. Prakasa Rao
Manager (OD), R & D, NMDC.

Question : Any attempt to design autogenous grinding mill ?

Author : The equations are developed only for ball mill and rod mill. We have not studied the design aspects for autogenous grinding mill.

V. Venkatachalam
Maharashtra Minerals Ltd , Bombay.

Question : What is the definition of unit 150 to 200.

Author : These figures relate to the mesh size. Ex: 200 mesh is equal to 74 micron (.074 mm). These are some standard screen/sieve sizes. There is nothing like 120 and 80 mesh in this series.