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MATHEMATICAL SIMULATION OF THE THERMAL FIELD DURING CONTINUOUS CASTING PROCESS BY A NONORTHOGONAL COORDINATE TRANSFORMATION FORMALISM

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ABSTRACT

A numerical technique, which includes a nonorthogonal coordinate transformation, a control volume based discretisation scheme and a method of search for determining the location of solid-liquid interface is developed for solving the thermal field during solidification process by continuous casting operation. The formulation is applied to continuous casting process of Al-Mg alloy cylindrical ingot. The theoretical prediction of the interface is in good agreement with the experimentally measured interface.

NOMENCLATURE

C	Specific heat
K	Thermal conductivity
T	Temperature
ρ	Density
U	Casting speed
x	Axial coordinate
r	Radial coordinate
$\vec{v}$	Velocity of the interface
R_o	Radius of the ingot
L_1	Mould height
L	Height of the solidified shell (Fig.1)
$\hat{i}$	Unit vector in the x-direction
h_1, h_2	Heat transfer coefficients
ξ	Transformed radial coordinate
η	Transformed axial coordinate
$\hat{n}$	Unit normal vector
∇	Vector differential operator
$\delta(r)$	Interface position
ΔH	Latent heat
Δ	Differential increment

Indices: l for liquid; s for solid

1. INTRODUCTION

After oxygen steel making, continuous casting is undoubtedly the most important technological advance in the metallurgical frontier during the post war period. It is used for metallurgical processing of materials and crystal growth. The importance of solidification process in metallurgical operations need not be overemphasized, as the metallurgical structure, distribution of inclusions, micro-and macro-segregation as well as metallurgical properties of the cast products are intimately associated with the solidification phenomena. Surface defects, gas blow holes and pin holes, shrinkage cracks, cavities and porosities are some of the typical defects generally associated with the process of solidification of metals and alloys. The heat transfer during solidification in a continuous casting process strongly influences product quality and process productivity. Therefore, theoretical prediction of the solidification front and the temperature field in ingots is of practical importance. A typical continuous casting process is shown in Fig. 1. The liquid metal with superheat is poured into the mould that has an internal passage through which the primary cooling water is circulated. The primary cooling water is jetted out of the bottom of the mould to impinge directly on the casting surface. Because of this heat removal from the

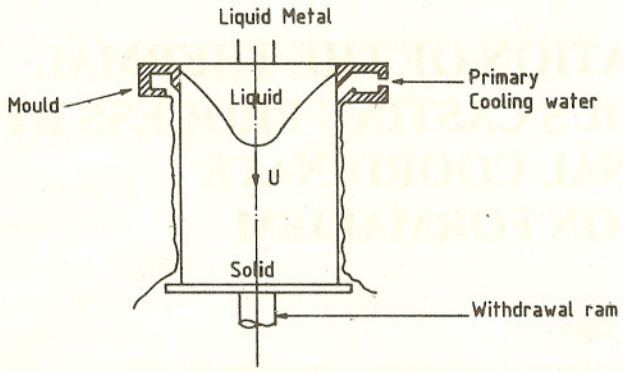

Fig. 1 : Continuous casting process.

molten metal, a solidified shell is formed. The shell must be sufficiently thick to contain the remaining liquid and permit continuous withdrawal of the casting.

During continuous casting process, the temperature distribution of ingots is independent of time, and only dependent of spatial coordinates. This amounts to a steady state thermal problem. An examination of the available literature indicates that the numerical techniques for solving the temperature field during solidification process can be classified into two distinct categories:

(a) The single region method, which applies the energy equation over the complete domain covering both of the phases. The latent heat release is simulated either by appropriate modification of specific heat or a scheme temperature rise.

(b) The multiple region method, which applies the governing equations separately for each phase and specifies the proper coupling boundary conditions between the phases. Since the position of the interface is a priori unknown and must be determined as part of the solution, this thermal problem can also be designated as free boundary problem.

In general, multiple region approach offers more accurate results that the single region approach[1]. Furthermore, it is more attractive because the multiple region method can be used to predict the effects of fluid flow in the melt or external factors such as rotations and electromagnetic stirring on solidification process. Practically, it is impossible for the single region method to attain these relatively complicated aims. Among the available literature for solving the temperature field problem during continuous casting by means of multiple region strategy, a conformal transformation of the solution domain was attempted to overcome the difficulties resulting from numerical analysis[2-4]. However, the conformal transformation is only applicable to the free boundary problems of two dimensions. Among the notable contributors in the field, Murray and Landis[1] used a variable space grid finite difference scheme for solving the moving boundary problems solidification. Gupta[5] used a Taylor expansion both in space and time to obtain functions values in successive time steps at points on grid system which moves bodily with the moving boundary. Miller et. al[6] studied the said Crank-Gupta oxygen consumption problem using finite elements in an adaptive mesh. Bonnerot and Jamet[7,8] used a variable space grid similar to that of Murray and Landis to construct isoparametric finite elements in space and time for non-rectangular grid. They have extended their method to two space dimensions. In one space dimension, various authors[9,10] have fixed the moving boundaries for all times by coordinate transformations. Such coordinate transformations bear the general idea of transforming a curved shaped region in two or more dimensions into a fixed rectangular domain. However for typical applications, the mapping functions are different depending upon the geometry and physics of the problem. The new curvilinear coordinates are referred to as bodyfitted coordinates. Following a pioneering paper by Winslow[11], successive authors have proposed various ways of using curvilinear grid. Useful lists of references are given by Thompson et al[12] and by Furzeland[13]. Oberkampf[14] discusses some useful generalized mapping function. Furzeland[13] has solved the problem of Bonnerot and Jamet using curvilinear transformations and compared his results with those obtained by other authors. Alternative mappings result from the use of finite elements and bivariate blending functions[15] or isoparametric curvilinear coordinates[16]. Saitoh[17] uses a version of the work of Duda et al[18]. Subsequently, Sparrow et al[19] have applied the same method in each phase of melting, initially subcooled region around a circular cylinder. Fundamental aspects of free and moving boundary problem have been elaborately described by Crank[20] in his excellent text. A method for handling multiple moving boundaries (in one and two dimensions) has been described in a recent paper by Kim and Kariany[21]. However, literature on

nonorthogonal transformations applied to continuous casting process of metals and alloys is rather scanty.

The objective of the present paper is to develop a numerical technique to effectively solve the thermal field problem of solidification in continuous casting in an efficient manner in two dimensions which can be extended to arbitrary dimensions. Consequently, as an application example of the present method, continuous casting of an Al-Mg alloy cylindrical ingot is considered in this study. Major emphasis has been given to the formalism of development of a numerical technique including a nonorthogonal coordinate transformation, a control volume based descretization scheme and a searching method for determining the interface position for studying the heat transfer phenomena during continuous casting. In the present study, a nonorthogonal transformation of an irregular region in physical space into a fixed rectangular region in computational space, which is applicable to free boundary problems of two and higher dimensions is employed to alleviate the difficulties arising from a priori unknown location of the solid-liquid interface. Then a control volume based discretization scheme is adopted to ensure energy conservation. Considerable amount of literature is available on the control volume based discretisation strategy for solving problems in heat, mass and momentum transfer[22–27]. This technique has been extensively used for the computation of flow, temperature and concentration fields in diverse industrial and physico-chemical processes. In many of the cases, orthogonal control volumes have been used for the discretisation of the governing transport equations. The basic concepts and methodology of control volume based discretisation has been described in the text of Patankar[28]. When the governing partial differential equations are integrated over discrete nonorthogonal control volumes, all terms arising from nonorthogonality of the grid are retained. In addition, since the location of the solid-liquid interface is a priori unknown, the problem is solved in an iterative fashion. In order to ensure that the iteration converges, a searching method similar to optimal algorithm has been developed for the determination of the interface. Finally, the thermal field of an Al-Mg alloy cylindrical ingot during continuous casting process has been obtained, and it is observed that the numerical prediction of the solid-liquid interface is in good agreement with the experimentally measured interface.

2. MATHEMATICAL FORMULATION

In modelling the heat transfer process in continuous casting shown in Fig. 1, for pure metals or short freezing range alloys, latent heat is considered to release along the solid-liquid interface; therefore, the present thermal problem is called the free boundary problem of two phases and two dimensions. Because of axial symmetry, the present problem becomes a two-dimensional one, which is applicable for all practical purposes. In addition, axial heat conduction is included in the analysis; liquid is assumed to move in slug flow and thermophysical properties of solid are different from those of liquid. The development of mathematical model for the continuous casting process can be depicted in the following manner. The governing transfer equations are (Fig.2),

$$\nabla . (K_l \nabla T_l) = C_l \rho_l U \frac{\partial T_l}{\partial x} \tag{1}$$

$$\nabla (k_s \nabla T_s) = C_s \rho_s U \frac{\partial T_s}{\partial x} \tag{2}$$

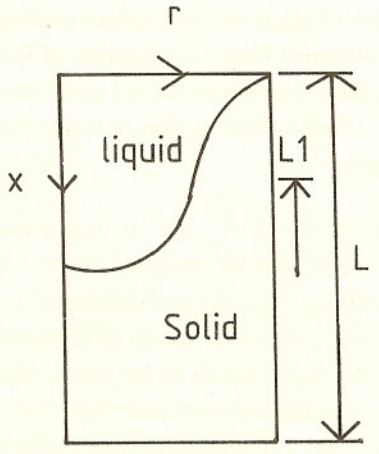

Fig. 2 : Solidification process model.

where, the subscripts l, s denote the liquidus and the solidus phase respectively. Boundary conditions at the solid-liquid interface are:

$$T_l = T_s = T_m \tag{3}$$

$$-K_s \frac{\partial T_s}{\partial n} + K_l \frac{\partial T_l}{\partial n} = \rho_s \Delta H (\vec{v} . \hat{n}) \tag{4}$$

where, T_m is the temperature of melting point, ΔH is latent heat, $\hat{n}$ is the unit normal vector from the melt at the interface, $\vec{v}$ is the velocity of the interface $\vec{v} = \hat{i}U$.

$$\text{At } x = 0 \text{ and } 0 \le r \le R_o, T = T_o \tag{5}$$

where, T_o is the casting temperature, R_o is the radius of the ingot.

$$\text{At } r = R_o \text{ and } 0 \le x \le L_1$$

$$-K_s \frac{\partial T}{\partial r} = h_1(T - T_{f1}) \tag{6}$$

where, L_1 is the mould height and T_{f1} is the temperature of the primary cooling water, h_1 is the effective heat transfer coefficient between surface of the ingot and primary cooling water.

$$\text{At } r = R_o \text{ and } L_1 \le x \le L,$$

$$-K_s \frac{\partial T}{\partial r} = h_2(T - T_{f2}) \tag{7}$$

where, T_{f2} is the average temperature of secondary cooling water, h_2 is the effective heat transfer coefficient between surface of ingot and secondary cooling water, which may be obtained from the equation of Rohsenow for nucleate boiling region with forced convection[32] and from the free falling turbulent film of water for forced convection region[33].

At $x = L$ and $0 \le r \le R_o$, if L is long enough, the bottom of the ingot can be assumed to be adiabatic; however, this will increase the computational load. If L is not long enough, an initial guess of temperature at the bottom of the ingot needs to be made, then these temperatures are adjusted to ensure that there are no discontinuities in the calculated thermal gradients near the bottom of ingot. An initial guess of the temperature is made in the present case.

$$\text{At } r = 0 \text{ and, } 0 \le x \le L, \quad \frac{\partial T}{\partial r} = 0 \tag{8}$$

The numerical solution procedure of the above coupled set of equations is complicated by the fact that the interface surface is a priori unknown and does not correspond to a coordinate surface under the framework of conventional cylindrical coordinate system.

3. COORDINATE TRANSFORMATION

The following nonorthogonal coordinate transformation has been employed in the present analysis. The salient features of the transformations are as follows.

In the melt; $0 \le \xi \le 1$, $0 \le \eta \le 1/2$

$$\text{where } \xi = r/R_o, \quad \eta = \frac{x}{2\delta(r)} \tag{9}$$

In the solid phase, $0 \le \xi \le 1$, $\frac{1}{2} \le \eta \le 1$

$$\xi = r/R, \quad \eta = 1 - \frac{L - x}{2(L - \delta(r))}$$

In computational space shown in Figs. 3(a) and 3(b) the solid-liquid interface is defined by the coordinate surface $\eta = 1/2$ and is fixed during iteration process of determining the interface position. This is highly desirable because it can both deal with curved irregular interface very well, and avoid the regeneration of mesh during iteration process. Consequently, the nonorthogonal coordinate transformation can overcome the fundamental difficulties from the a priori unknown location of the solid-liquid interface, such that the complexity of numerical analysis and algorithm development can be greatly reduced.

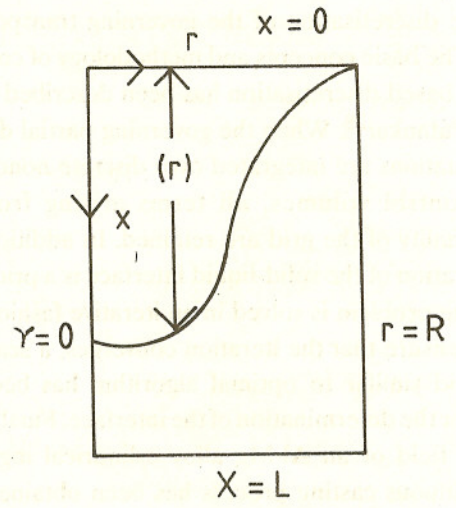

Fig. 3(a) : Solution domain in physical space.

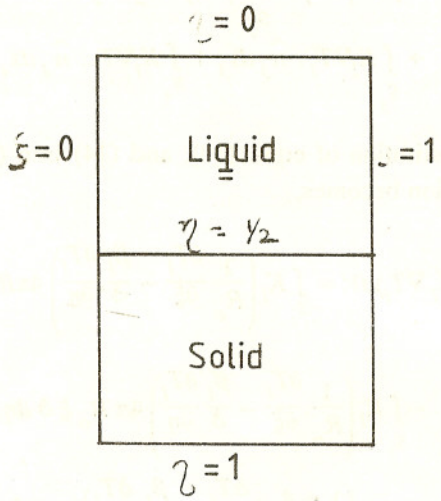

Fig. 3(b) : Solution domain in computational space.

Using the chain rule of differentiation of a function, following relations are derived which will be of subsequent use. In melt,

$$\frac{\partial}{\partial x} = \frac{1}{2\delta}\frac{\partial}{\partial \eta} \tag{10.1}$$

$$\frac{\partial}{\partial r} = \frac{1}{R_o}\frac{\partial}{\partial \xi} - \frac{\eta}{R_o \delta}\frac{d\delta}{d\xi}\frac{\partial}{\partial \eta} \tag{10.2}$$

and the vector differential operator is given as

$$\nabla = \left(\frac{1}{R_o}\frac{\partial}{\partial \xi} - \frac{\beta_l}{\partial}\frac{\partial}{\partial \eta}\right)\hat{r} + \left(\frac{1}{2\delta}\frac{\partial}{\partial \eta}\right)\hat{i} \tag{10.3}$$

where, $\beta_l = \frac{\eta}{R_o}\frac{d\delta}{d\xi}$, $\hat{r}$ and $\hat{i}$ are unit vectors in radial and axial directions respectively. In solid,

$$\frac{\partial}{\partial x} = \frac{1}{2(L-\delta)}\frac{\partial}{\partial \eta} \tag{11.1}$$

$$\frac{\partial}{\partial r} = \frac{1}{R_o}\frac{\partial}{\partial \xi} - \frac{(1-\eta)}{R_o(L-\delta)}\frac{d\delta}{d\xi}\frac{\partial}{\partial \eta} \tag{11.2}$$

$$\nabla = \left(\frac{1}{R_o}\frac{\partial}{\partial \xi} - \frac{\beta_s}{(L-\delta)}\frac{\partial}{\partial \eta}\right)\hat{r} + \left(\frac{1}{2(L-\delta)}\right)\frac{\partial}{\partial \eta}\hat{i} \tag{11.3}$$

where, $\beta_s = \frac{1-\eta}{R_o}\frac{d\delta}{d\xi}$

4. CONTROL VOLUME DISCRETIZATION FORMALISM

The most attractive feature of control volume based discretization scheme is that the resulting solution would imply that the integral conservation of quantities such as mass, momentum and energy is exactly satisfied over any group of control volumes and, of course, over the whole calculation domain. Furthermore, this characteristic also exists even if the mesh size is large[28,29]. In order to obtain the discretized energy equations, the continuum calculation domain is first divided into discretized nonorthogonal control volumes. For a nonorthogonal control volume shown in Fig. 4, the infinitesimal element volume within control volume and infinitesimal element areas at control volume faces are

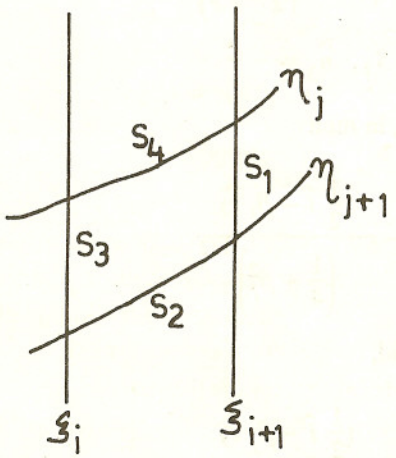

Fig. 4 : Non-orthogonal control volume.

as follows:

In melt,

$$dV = 4\pi R_o^2 \xi\delta\, d\xi\, d\eta \tag{12.1}$$

$$dS_1 = dS_3 = 4\pi R_o \xi\delta\, d\eta \tag{12.2}$$

$$dS_2 = dS_4 = 2\pi R_o^2 \xi\,(1 + 4\beta_l^2)^{1/2}\, d\xi \tag{12.3}$$

In solid,

$$dV = 4\pi R_o^2 \xi\,(L-\delta)\, d\xi\, d\eta \tag{13.1}$$

$$dS_1 = dS_3 = 4\pi R_o^2 \xi\,(L-\delta)\, d\eta \tag{13.2}$$

$$dS_2 = dS_4 = 2\pi R_o^2 \xi\,(1 + 4\beta_s^2)^{1/2}\, d\xi \tag{13.3}$$

The outward unit normal vectors at control volume faces are;

$$\text{At faces } S_1, \quad \hat{n}_1 = \frac{\nabla \xi}{|\nabla \xi|} = \hat{r} \tag{14.1}$$

At face S_2, in melt,

$$\hat{n}_2 = \frac{\nabla \eta}{|\nabla \eta|} = \frac{\frac{1}{2}\hat{i} - \beta_l \hat{r}}{\left(\frac{1}{4} + \beta_l^2\right)^{1/2}} \tag{14.2}$$

and in solid,

$$\hat{n}_2 = \frac{\nabla \eta}{|\nabla \eta|} = \frac{\frac{1}{2}\hat{i} - \beta_s \hat{r}}{\left(\frac{1}{4} + \beta_s^2\right)^{1/2}} \tag{14.3}$$

$$\text{at face } S_3, \quad \hat{n}_3 = -\hat{r} \tag{14.4}$$

at face S_4, in melt

$$\hat{n}_4 = -\frac{(\frac{1}{2}\hat{i} - \beta_l \hat{r})}{\left(\frac{1}{4} + \beta_l^2\right)^{1/2}} \tag{14.5}$$

and in solid,

$$\hat{n}_4 = -\frac{(\frac{1}{2}\hat{i} - \beta_s \hat{r})}{\left(\frac{1}{4} + \beta_s^2\right)^{1/2}} \tag{14.6}$$

The, thermal transport (energy) equations are integrated over each nonorthogonal control volume in physical space. For melt region, the expression will be:

$$\int_V \nabla . (K_l \nabla T_l) dV = \int_V C_l \rho_l U \frac{\partial T_l}{\partial x} dV \tag{15}$$

with application of Gauss theorem, the left hand side of eqn. (15) becomes,

$$\int_V \nabla . (K_l \nabla T_l) dV = \int K_l (\nabla T_l . \hat{n}) ds$$

$$= \int_{S_1} \nabla T_l . \hat{n}_1 \, ds_1 + \int_{S_2} K_l \nabla T_l . \hat{n}_2 \, ds_2 + \int_{S_3} K_l \nabla T_l . \hat{n}_3 ds_3 + \int_{S_4} K_l \nabla T_l . \hat{n}_4 \, ds_4 \tag{16}$$

On substitution of eqns. (10), and (14) into (16), the expression becomes,

$$\int_V \nabla . (K_l \nabla T_l) dV = \int_{S_1} K_l \left(\frac{1}{R_o} \frac{\partial T_l}{\partial \xi} - \frac{\beta_l}{\delta} \frac{\partial T_l}{\partial \eta} \right) 4\pi R_o \xi \delta d\eta$$

$$- \int_{S_3} K_l \left(\frac{1}{R_o} \frac{\partial T_l}{\partial \xi} - \frac{\beta_l}{\delta} \frac{\partial T_l}{\partial \eta} \right) 4\pi \, R_o \, \xi \, \delta \, d\eta$$

$$+ \int_{S_2} K_l \left(\frac{1+4}{4\delta} \beta_l^2 \frac{\delta T_l}{\delta \eta} - \frac{\beta_l}{R_o} \frac{\delta T_l}{\delta \xi} \right) 4\pi \, R_o^2 \, \xi \, d\xi$$

$$- \int_{S_4} K_l \left(\frac{1+4}{4\delta} \beta_l^2 \frac{\delta T_l}{\delta \eta} - \frac{\beta_l}{R_o} \frac{\partial T_l}{\partial \xi} \right) 4\pi \, R_o^2 \, \xi \, d\xi \tag{17}$$

The right hand side of eqn. (15) can be expressed as,

$$\int_V C_l \rho_l U \frac{\partial T_l}{\partial x} dV = 2\pi R_o^2 \, C_l \rho_l U \int \frac{\partial T_l}{\partial \eta} \xi \, d\xi \, d\eta \tag{18}$$

On substitution eqns (17) and (18) into eqn (15), the following thermal transport (energy) equation in differential-integral form in $(\xi - \eta)$ coordinate system is derived.

$$\int_{S_1} K_l \frac{\partial T_l}{\partial \xi} d\xi \, d\eta - \int_{S_3} K_l \frac{\partial T_l}{\partial \xi} d\xi \, d\eta$$

$$+ \int_{S_2} K_l \left(\frac{1+4}{4\delta} \beta_l^2 \right) \frac{\partial T_l}{\partial \eta} R_o^2 \, \xi \, d\xi$$

$$- \int_{S_4} K_l \left(\frac{1+4}{4\delta} \beta_l^2 \right) \frac{\partial T_l}{\partial \eta} R_o^2 \, \xi \, d\xi = \int_{S_1} K_l \beta_l \frac{\partial T_l}{\partial \eta} R_o \, \xi \, d\xi$$

$$- \int_{S_3} K_l \beta_l \frac{\partial T_l}{\partial \eta} R_o \, \xi \, d\eta + \int_{S_2} K_l \beta_l \frac{\partial T_l}{\partial \xi} R_o \, \xi \, d\xi$$

$$- \int_{S_4} K_l \frac{(1 + 4\beta_l^2)}{4\delta} \frac{\delta T_l}{\partial \eta} R_o^2 \, \xi \, d\xi$$

$$= \int_{S_1} K_l \beta_l \frac{\partial T_l}{\partial \eta} R_o \xi \, d\eta - \int_{S_3} K_l \beta_l \frac{\partial T_l}{\partial \eta} R_o \xi \, d\eta$$

$$+ \int K_l \beta_l \frac{\partial T_l}{\partial \xi} R_o \xi \, d\xi - \int_{S_4} K_l \beta_l \frac{\partial T_l}{\partial \xi} R_o \xi \, d\xi$$

$$+ \frac{R_o^2}{2} C_l \rho \, U \int_V \frac{\partial T_l}{\partial \eta} \xi \, d\xi \, d\eta \qquad (19)$$

It may be observed that the first four terms in the right hand side of the above equation result from the nonorthogonality of grids. The equation in its differential-integral form has been discretized in the computational space of which the grids are straight orthogonal (Fig. 5). Certain rules are observed while discretizing eqn. (19)

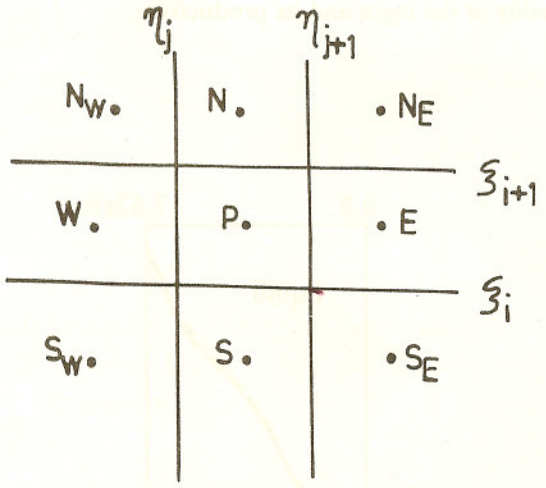

Fig. 5 : Computational space with orthogonal grids.

i.e, the integrated function of each surface integral term is replaced with the function value at a node point P within the control volume. Consequently, following discretized energy equation in the melt has been obtained.

$$0 \le \xi \le 1, \quad 0 \le \eta \le 1/2$$

$$a_p T_p = a_N T_N + a_s T_s + a_E T_E + a_W T_W + S_c \qquad (20)$$

It should be stated that in order to make the discretized equation be in the form of eqn. (20), which is easy to solve, all the terms related with the values of temperature at node points N_W, N_E, S_W, S_E, which emerges from the nonorthogonality of the grid, are incorporated into the source term S_c in eqn. (20) and the current values are adopted in the computation process. In addition, the coefficients a_p in eqn. (20) must be positive so as to assure that iteration process converges. The upwind scheme has been introduced in discretization of the $\frac{\partial T}{\partial \eta}$ of volume integral $\frac{\partial T}{\partial \eta} = \frac{T_p - T_w}{\eta_p - \eta_w}$. Likewise, the discretized energy equation in solid phase can be derived in $(\xi - \eta)$ coordinate system,

$$0 \le \xi \le 1, \quad 1/2 \le \eta \le 1$$

$$a_p T_p = a_N T_N + a_s T_s + a_E T_E + a_W T_W + S_c \qquad (21)$$

along with the associated boundary conditions.

5. SOLUTION PROCEDURE

The free boundary problem of temperature field during continuous casting has to be solved in an iterative fashion. The general iterative solution procedure will be as follows.

(i) A solid-liquid interface position $\delta(r)$ is assumed.

(ii) Solution is sought for the discretized energy equation in the melt and solid region respectively with a line by line iterative scheme.

(iii) An improved interface position is obtained by satisfying the energy balance equation at solid-liquid interface with the temperature gradient from step (ii)

(iv) Iterative sequence from step (ii) is repeated until convergence has been achieved.

The energy balance equation at the solid-liquid interface in $(\xi - \eta)$ coordinate system has been given as:

$$\left[1 + \left(\frac{1}{R_o} \frac{d\eta}{d\xi}\right)^2\right] \left[\frac{K_l}{2\delta} \left(\frac{\partial T_l}{\partial \eta}\right)_{\eta=\frac{1}{2}} - \frac{K_s}{2(L-\delta)} \left(\frac{\partial T_s}{\partial \eta}\right)_{\eta=\frac{1}{2}}\right]$$

$$= \rho \Delta H U \qquad (22)$$

The discretized form of eqn. (22) is a set of nonlinear algebraic equations. Though, a set of nonlinear algebraic equations can be solved by some iteration scheme, it is only when initially guessed values approach it's solution

closely. Otherwise, the iteration process would diverge or converge to a physically unrealistic result. Therefore, in this present work, a searching method similar to optimal algorithm has been developed for determining improved interface position.

During searching process, the location of solid-liquid interface is moved. If heat loss from the interface at node i is greater than the sum of heat input to the interface at node i plus the latent heat released here, the interface at node i needs to be moved a distance in positive x- direction. Once, the energy balance requirement is satisfied, the interface will be stationary. At the same time, the searching iteration procedure may also be regarded as pseudo-transient, where the interface at node i moves with a velocity W_i in x- direction and its one iteration is equivalent to a time interval, the pseudo transient process elapses. Therefore, the expression for interface location of the k-th iteration can be represented as,

$$\delta_i^k = \delta_i^{k-1} + W_i \Delta t \tag{23}$$

$$W_i = U \left[1+\left(\frac{1}{R_o}\frac{d\delta}{d\xi}\right)_i^2\right]\left[\frac{K_l}{2\delta_i}\left(\frac{\partial T_l}{\partial \eta}\right)_{\eta=\frac{1}{2}} - \frac{K_s}{2(L-\delta_i)}\left(\frac{\partial T_s}{\partial \eta}\right)_{\eta=\frac{1}{2}}\right] \Big/ \rho \Delta H \tag{24}$$

When all $W_i \to 0$, the interface would attain its steady state position and the energy balance requirement will be satisfied.

6. APPLICATION EXAMPLE

In order to verify the application of the formalism, continuous casting of a 0.1524 m diameter A6063 Al-Mg alloy cylindrical ingot[30] has been taken as an example for simulation. The casting speed is 3.80 x 10^{-3} m.s^{-1} and the pouring temperature of the metal is 963 K. In addition, the initial cooling water temperature of the liquid metal is constant at 280 K, the flow rate is 1.89 x 10^{-3} m^3.s^{-1} and the average temperature of the secondary cooling water is 303 K. The freezing range of A6063 Al-Mg alloy is narrow enough that it can be modelled as zero freezing range alloy is narrow enough that it can be modelled as zero freezing range alloy with melting point of 923 K and the latent heat is released along the interface. Therefore, this steady state thermal field problem. A computational algorithm has been devised to solve the problem in the light of the numerical method. The thermal profile in the melt and the solid, and the location of solid-liquid interface have been obtained. Computational result shows that the theoretical prediction of the interface is in good agreement with the experimentally measured interface as reported by Weckman and Niessen[30]. Figure 6 shows the comparison of these profiles. This is an indication of the prediction credibility of the numerical formalism. Figure 7 shows some computed isotherms during continuous casting process. Some important parameters such as casting speed, secondary cooling conditions will influence the interface shape and further influence quality of the ingot and its productivity.

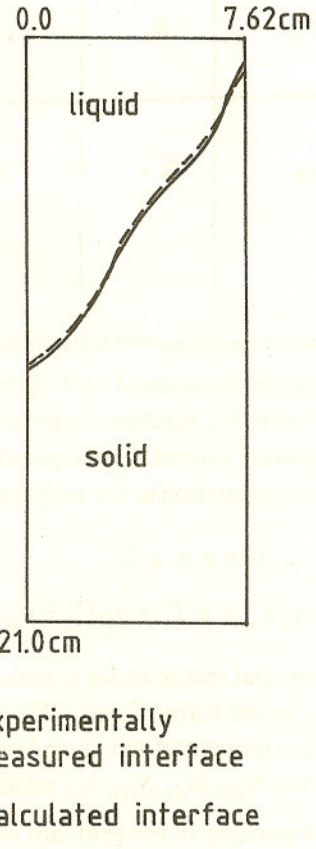

Fig. 6 : Solidification profile of the ingot.

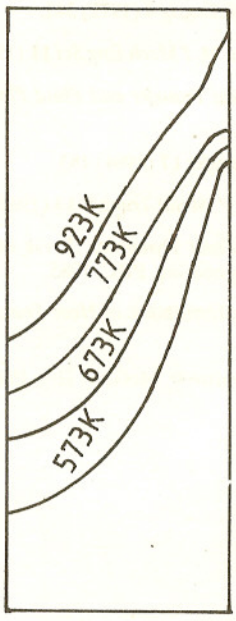

Fig. 7 : Thermal profile distribution.

7. CONCLUSION

The essence of the present study is the development of a numerical formalism which comprises of a nonorthogonal coordinate transformation, a control volume based discretization procedure and a searching method for the determination of the location of the solid-liquid interface for solving temperature field during solidification in continuous casting. The formalism can deal with the irregular shape of the interface well, avoid the generation of mesh during iteration and ensure the iteration to converge. In addition, all the terms arising from the nonorthogonality of the grid are retained in the solution procedure. It is expected that the numerical formalism can be utilized to solve the free boundary of three dimension and with further modification, it can take into account the effects of fluid flow in the melt and external factors which as rotation and electromagnetic stirring on solidification process. As a verification application, theoretical simulation of continuous casting of Al-Mg alloy cylindrical ingot has been carried out with axial heat conduction. The predicted profile has been compared with the experimental data and found to be in good agreement. A comparison of the present work with the recent work of Kim and Kaviany[21] can also be described in few lines. They have developed a finite difference based method on coordinate transformation to tackle phase change problems with moving boundaries of irregular shape. The discretised conservation equations are associated with moving control volumes which undergoes stretching/contraction in the physical coordinates. The moving boundaries are treated explicitly to avoid iterations, while the temperature field equations are treated implicitly. The method is not applicable for problems where latent heat is released over a range of temperature. In the present work a searching technique has been developed for iterative convergence of the solution algorithm to a physically realistic result. At the same time the searching iteration procedure may be regarded as pseudo-transient, where the interface moves in the axial direction and it's one iteration is equivalent to a time interval the pseudo-transient process elapses. The location of the interface is computed in an efficient manner at every iteration.

Basing upon the theoretical simulation of the temperature field during solidification in continuous casting, it is possible to predict the effects of some technological parameters on the solidification and crystal growth by a heat transfer analysis which can be the scope of future work.

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