FINITE ELEMENTS MODELLING OF FORMING PROCESSES A STATE OF THE ART REVIEW

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Traditionally metal forming processes have attracted attention of investigators interested in developing analytical models of metal working processes. Upper bound and lower bound techniques are the results of such efforts. However, the shortcomings of these techniques such as the capability to get limited information (i.e. approximate determination of load and power only) and application to generally axi-symmetric cases alone remained major limitations. This remained the only analytical method available to the users of metal forming processes till late sixties and early seventies [1]. The situation started changing in early seventies when use of numerical techniques became common even for complex problems because of the availability of high speed computers. A very strong numerical technque, finite elements method, came into prominence and attempts were made to use it to solve the traditiohal metal forming problems of forward and backward extrusion [2,3,4]. This paved the way for extracting much more useful information from mathematical solution such as strain rate distribution, knowledge about total plastic strain for indentifying possible locations of fracture beaides the conventional results of extrusion load and power. Attempts were also made to analyze temperature distribution and assess its effect in hot metal-working processes [5]. Such thermal effects are impossible to analyze using the conventional upper bound and lower bound techniques . Since then the progress has been fairly rapid in the application of finite element technique to analyze various metal forming processes. Eighties saw great strides in the application of FEM in metal forming. This method has been used to analyze extrusion [6], slab rolling [7,8,9], strip rolling [10], forging [11,12], stretch forming [13] etc. The recent approach is to integrate several different effects in single solution such as heat flow, stress analysis etc., where one affects the second and vice versa. Such coupled analyses have lately been reported and applied to metal forming problems [6,14].

INTRODUCTION TO FEM

The finite elements method involves dividing the whole zone of analysis into a large number of small sub-dividions, known as elements. Elements may be of different shapes but generally triangular elements are chosen for two dimensional body having uniform thickness and tetrahedron elements are used for three dimensional objects. The size of elements may vary within the body with small elements being used in regions of high stress or temperature variation so that desired accuracy can be obtained in the solution. The corners of the elements are called modes and the values of the parameter (say temperature in heat flow analysis and displacement in stress analysis) at the nodes are the unknown variables of the problem. Hence, there will be as many unknowns as the number of nodes or if there exist more than one unknown at a node (say displacements in x-and y directions) the total number of unknowns will be appropriate multiple of the number of nodes. To solve the problem, the governing equation/s (which may be heat conduction equation or force equilibrium equations) is satisfied at each node using any one of the several techniques available. Thus consideration of all the nodes in this manner results in as many linear equations as the number of unknowns at the nodes. Any method available for solving a set of simultaneous equations can be used to solve these equations to get results.

The problem of computing coefficients for such a large number of equations and solving these can be tackled only by means of a fast computer with sufficient memory. The advantage of the method lies in the fact that different material properties can be considered in different elements depending upon the situation prevailing there. Say for the case of elasto-plastic thermal stress analysis the yield point of the material will depend upon temperature which will be different in various elements. The versatility of the method makes it suitable for application in many areas such as structural mechanics, dynamic heat flow by conduction and convection (especially cooling of castings and weldments), fluid flow, plastic flow of metals and polymers (metal forming), electric and magnetic field distribution, diffusion of material etc. The governing equation/s of the phenomenon are to be written in finite element form, which is not often difficult.

The procedure for general finite elements formulation is explained in appendix I with the example of 2 dimensional stress analysis using triangular elements.

APPLICATION TO METAL FORMING

Different metal forming processes pose the problem for mathematical formulation in different ways. Thus the various approaches used for solving these various forming problem differ somewhat from each other. The problems of hot or cold extrusion or rolling of slabs or strips generally involve perfectly plastic behaviour of the material when it passes through the extrusion dies or rolls in rolling operation. A rigid-perfectly plastic material behaviour may be assumed. Also the conditions of stress and strains over a point, located with reference to origin at die or roll, remain same throughout the operation. It can be visualized as if a viscous fluid is flowing through the die or rolls under steady state condition. Similarity between the flow of viscous fluid and such metal forming operations is exploited here and such formulation is known as "flow formulation". In hot forging operations in open or closed dies again the material may be assumed to behave like rigid-perfectly plastic material. However, the condition are not in a steady state and the shape of the preform is charging continuously during the operation. Unsteady state "flow formulation" can be used for such cases. In operations like press working or stretch forming, the various regions of strip may be under different states of elasto-plastic strains However, the displacements are very large compared to dimensions (thickness) of the strip. Here elasto-plastic analysis is to be carried out but the theory of infinitesimal strains cannot be used and appropriate large displacement formulation for strains is to be used. This approach has sometimes been applied to processes such as extrusion and rolling also. Thus basically there are two approaches for analyzing stresses and strains in metal forming operations.

(1) Flow formulation

(2) Large displacement strain-formlulation
(or geometrically non-linear problem)

For analyzing temperature distribution, unsteady or steady state heat conduction equations are solved using Euler's theorem of variational calculus or weighted residue approach with Galerkin process.

The formulation for these cases are explained in standard text books [15]. A problem which is faced in the applications of these methods is the consideration of temperature dependent or strain rate dependent metal properties. Thermal conductivity, specific heat, yield point are the examples of temperature dependent metal properties while effective coefficient of viscosity for metal flow is the example of strain rate dependent property generally encountered when flow formulation or even large strain formulation is used. Consideration of such properties demands number of iterative solutions at every step so that desired variation in properties can be suitably accounted. Methods are used to reduce the number of iterative steps so as to reduce computational time and keep it with in reasonable limit. Newton-Raphson method; suitably developed for matrices, is generally used for this purpose. Yet, the number of steps and total computation time becomes quite large,as will be seen from the examples given here.

EXAMPLES

A few examples of finite elements method to various metal forming operations are reported here which are taken from recent literature.

1. Plate Rolling

The problem of hot rolling of plate has been analyzed by several investigators [7,8,9]. In some of these [7] the thermo-mechanical coupling is considered where the strain energy of plastic flow is considered as an additional source of heating. The analysis of plastic strain has been carried out [9] and effective plastic strain in different region of plate is reported. The author also points out that the total equivalent plastic strain is an important variable for tracking the microstructural evolution of steel in the plate. For analysing single set of parameters, the number of iterative steps reported are 99 to 205 and the actual computation time on a fast computer has been reported to vary between 3.5 and 7 hrs. for each set. Some of the results reported there are shown in figure 1. The amount of information available from these solutions is obviously quite appreciable which can be used for arriving at important mechanical and metallurgical conclusions in a logical manner.

2. Strip Rolling :

The distortion of thin strips is a problem during the rolling operation and this has been analyzed by Yukawa et. al. [10] using large displacement strain formulation and finite element method. Some of the results are reported in figure 2. He has analyzed stresses in the two cases of flat rolls and crowned rolls, which show different distorted shapes and distribution of residual stresses. The effect of the magnitude of residual rolling stresses has also been analyzed to show that for some thereshold distribution of redisual stress no distortion accurs in the rolled plates.

3. Stretch Forming

Stretching and large deflection of strip generally occurs during drawing operation when the parts are fabricated by press working. The problems like excessive strains and fracture and spring back are common in such operations. FEM has been used to analyse such problem and the results are reported [13] for simple square punch forming as well as forming of complex oil-pan of automobile in aluminium alloy. The places where fracture is expected to occur are identified on the basis of strain distribution. Experimental observations have shown that these were the places where fracture actually occured. figure 3 shows some of the results reported. The method has used large deformation stress formulation with consideration of friction and work hardening.

4. Forging

Several investigators have reported finite elements formulation for forging operation [12,14]. The example reported here is that of forging of an automobile differential Crown [12]. The method again uses large deformation strain formulation. The friction between die and preform is considered. A special algorithm, using gap elements, has been developed to detect the instant when the metal being forged makes contact with the die surface. At this instant boundary constraints start applying on the forging preform. Figure 4 shows various stages during forging of this component as analyzed using the method proposed. Quadrilateral axisymmetric ring type elements are used. Only one half of the cross section is shown in the figure since the other half is symmetrical. The top die was moving down with a velocity of 226 mm/s. A non-work hardening elastic-plastic material was assumed. Friction was applied using the cohesive model with a constant friction factor of m = 0.15 on both dies. As the top die moves down, the state of stress and plastic strain in all the elements was determined after each small increment of time. The first set contained 116 elements and initial time increment considered was 0.0039 sec. After 20 such increments the state of forging is shown at (c) in the figure. At this state some of the elements have become highly distorted so remeshing was done and number of elements increased to 224. This is shown at (d). After 15 more steps, the deformed mesh is shown at (e). Remeshing was done again and the time step now reduced to 0.00195 sec. The number of elements increased to 294 introducing more elements at the point where flash formation started. The state of forging after a total of 56 steps is shown at (g) in the figure. The final position of dies and forging after next 5 steps of 0.001 sec. is shown at (h). The final distribution of plastic strain is shown at (k). Regions of large localised strains are very obvious in the figure.

FEM PACKAGES

There are a number of finite element packages available. Traditionally, the packages have placed more emphasis on the development and formulation for elements used more commonly for stress analysis in structures. Few of these also provide formulation for heat conduction analysis and also fluid flow analysis. Provisions for considering large deformation strain formulation may also be present. More elaborate packages have provision for considering plastic behaviour of metal and also the temperature dependent material properties. Modelling for coupled analysis is a recent phenomena and these provisions may not be available in the packages. However, packages do help the investigators by saving him from lot of time consuming hard work and frustation inherent in developing his own programme. In the examples shown above the investigators have generally used the packages developed by themselves.

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APPENDIX

Formulation for Finite Elements Analysis of 2D Triangular Element

Figure 5 shows a triangular element drawn in x-y Cartesian coordinate system. The coordinates of the three nodes i, j and m are x_i, y_i .. etc. Small displacements of the nodes due to straining of the element are designated as u_i , v_i ... etc. and the corresponding forces acting at the nodes due to adjoining elements or from external sources are designated as U_i , V_i etc. An assumption is made that the displacements u, v vary linearly within the element, which is a reasonable assumption when the size of triangular element is small (many other types of non linear variation of u, v within the element have also been analyzed). The general expressions for displacements u, v within the element can be written in terms of nodal displacements u_i , v_i etc. First the expressions for u, v are written in terms of coefficients ,α_1,α_2 ... α_6 and then these coefficients are determined to give expressions for u, v in terms of nodal coordinates and nodal displacements, as weitten in figure 5. It is convenient to write these expressions in vector and matrix form. The vector $\{a^e\}$ is elemental displacement vector having three component vectors $\{a_i\}$,$\{a_j\}$ and $\{a_m\}$ at the three nodes which themselves have two displacement components as shown in the figure. The relationship between general displacement vector and nodal displacement vectors is written in matrix notations as

$$\{u'\} = [N]\{a^e\}$$

The matrix [N] is known as shape function. The strain vector is designated by $\{\varepsilon\}$ infigure 6 and it is written in terms of operator matrix [L] and displacement vector $\{u'\}$. It should be easy to understand because strain is defined in terms of differentials of displacements. The expression can be written in more compact form in terms of matrix [B] and vector $\{a^e\}$ as shown. Since stresses and strains are related through the material properties (E, Youn-g's modulus and , ν Poisson's ratio), the expression for stress vector $\{\sigma\}$ can be derived using the individual relation between ε_x, σ_x etc. as written in figure 6. Thus stress vector$\{\sigma\}$ can be written in more compact form in terms of elasticity matrix [D] and strain vector $\{\varepsilon\}$.

On writing down the expression for potential energy over the whole traingle for linear variation of stress with strain we get,

$$P.E = \frac{1}{2}\int_A \{\varepsilon\}^T\{\sigma\}t\ dA\ ,$$

where the integral is to be taken over the whole area, A, of the traingle . 't' is the thickness of the element. It can be rewritten in the form

$$P.E = \frac{1}{2}\int_A \{\varepsilon\}^T[D]\{\varepsilon\}t\ dA$$

$$= \frac{1}{2}\{a^e\}^T\int_A [B]^T[D][B]t\ dA.\{a^e\}$$

($\{a^e\}$ being constant over the area of integration A)

On adding such expressions for all the elements (n) and then minimizing the total potential energy (including the work done against the external forces) with respect to nodal displacement, we get the final expression as

$$[K]\{a\} = \{R\} \longrightarrow (1)$$

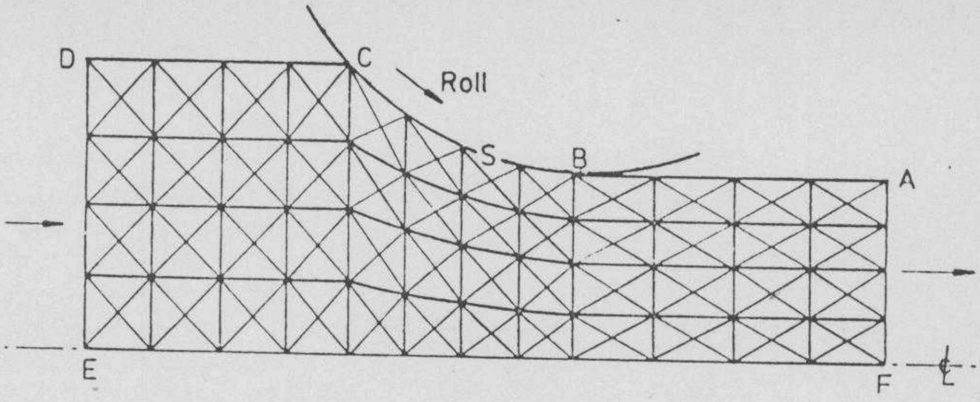
$$\text{where } [K] = \sum_{e=1}^{n} [k^e]$$

$$\text{and } [k^e] = \int_A [B]^T[D][B]\, t\, dA \qquad \text{for element 'e'}$$

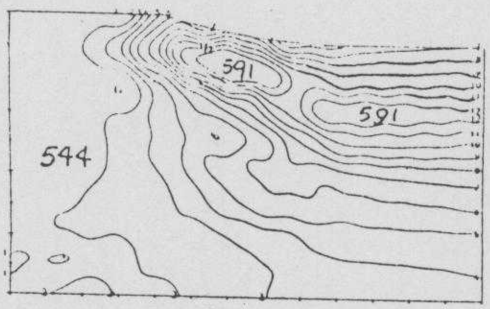
$\{R\}$ is the vector designating the external forces acting at the nodes and $\{a\}$ is the vector comprising the displacements at all the nodes (m) i.e.

$$\{a\} = \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_m \end{Bmatrix} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ \vdots \\ u_m \\ v_m \end{Bmatrix} \longrightarrow (2)$$

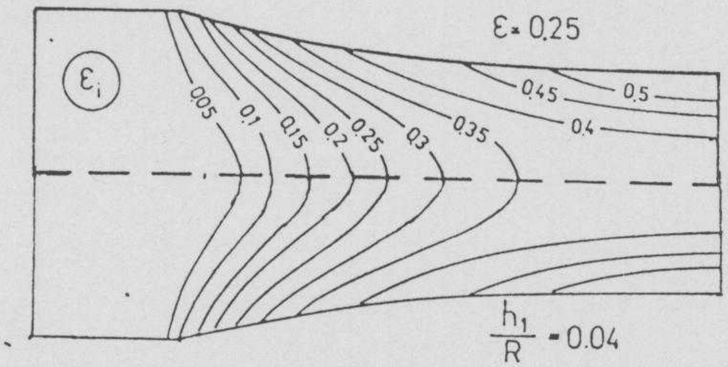
Equation (1) is thus a set of "2m" simultaneous equations in terms of 2m variables shown in eg. (2), (for 2 dimensional case). These equation can now be solved to get the values of u_1 , v_1... u_m , v_m at all the nodes. Any standard technique available for matrix inversion can be used. The displacement u_i, v_i.... etc. so determined can be used to determine stresses etc. in all the elements using the expressions developed earlier and shown in figure 6. The above treatment is fairly simplified and the general expression will contain several other vector terms as shown in figure 6.



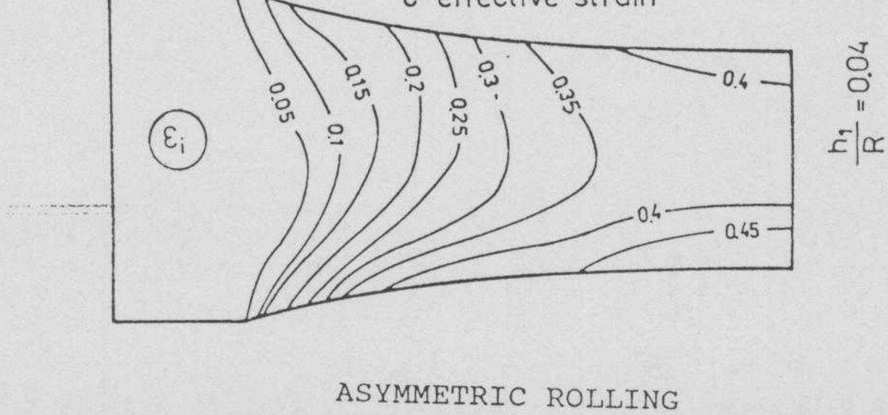
FEM GRID



TEMPERATURE ISOTHERMS



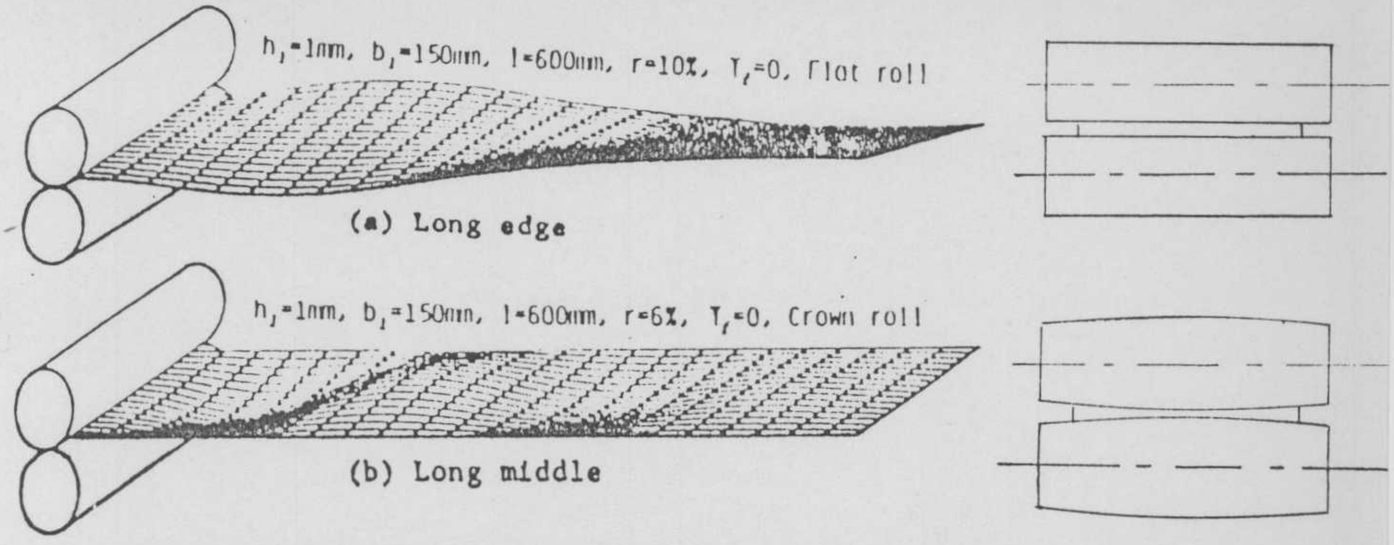
EFFECTIVE STRAIN   SYMMETRIC ROLLING



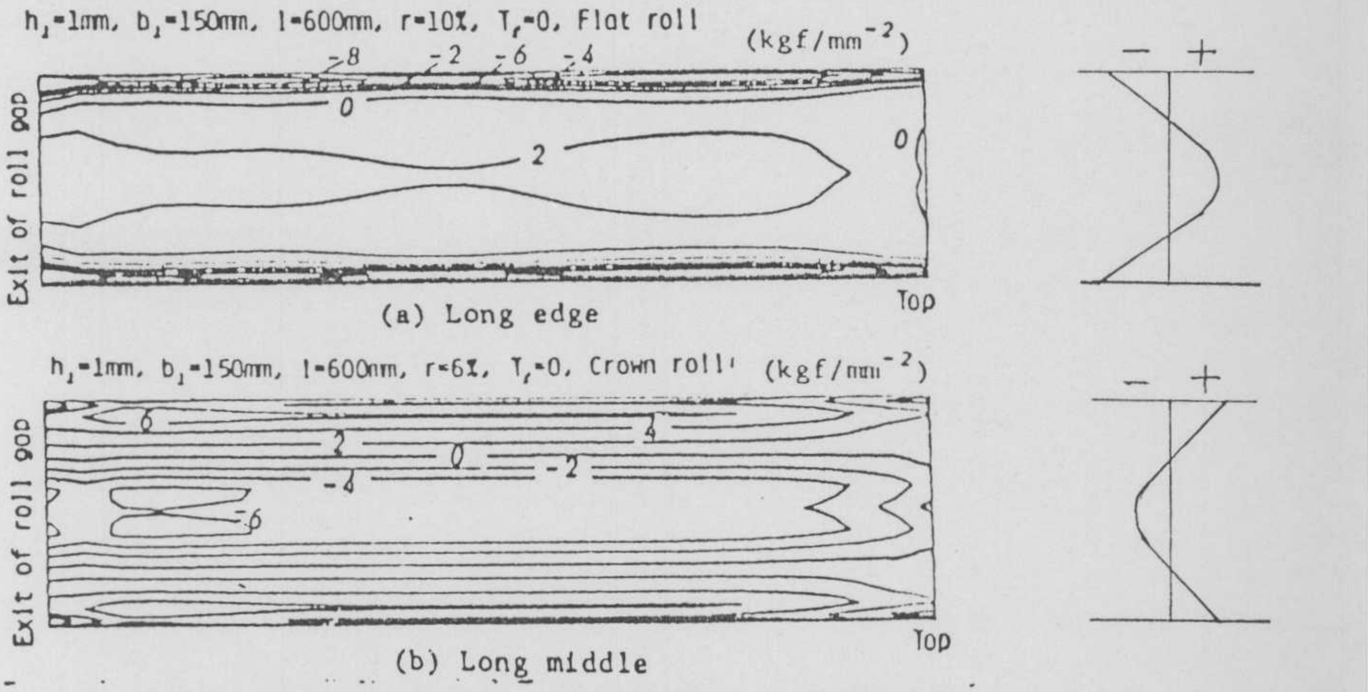
ASYMMETRIC ROLLING

BEYNON ,J.H.et.al. 1986 ,   PIETRZYK ,M., 1986
GROBER ,H., 1986

Fig.1 - Analysis of Plate Rolling



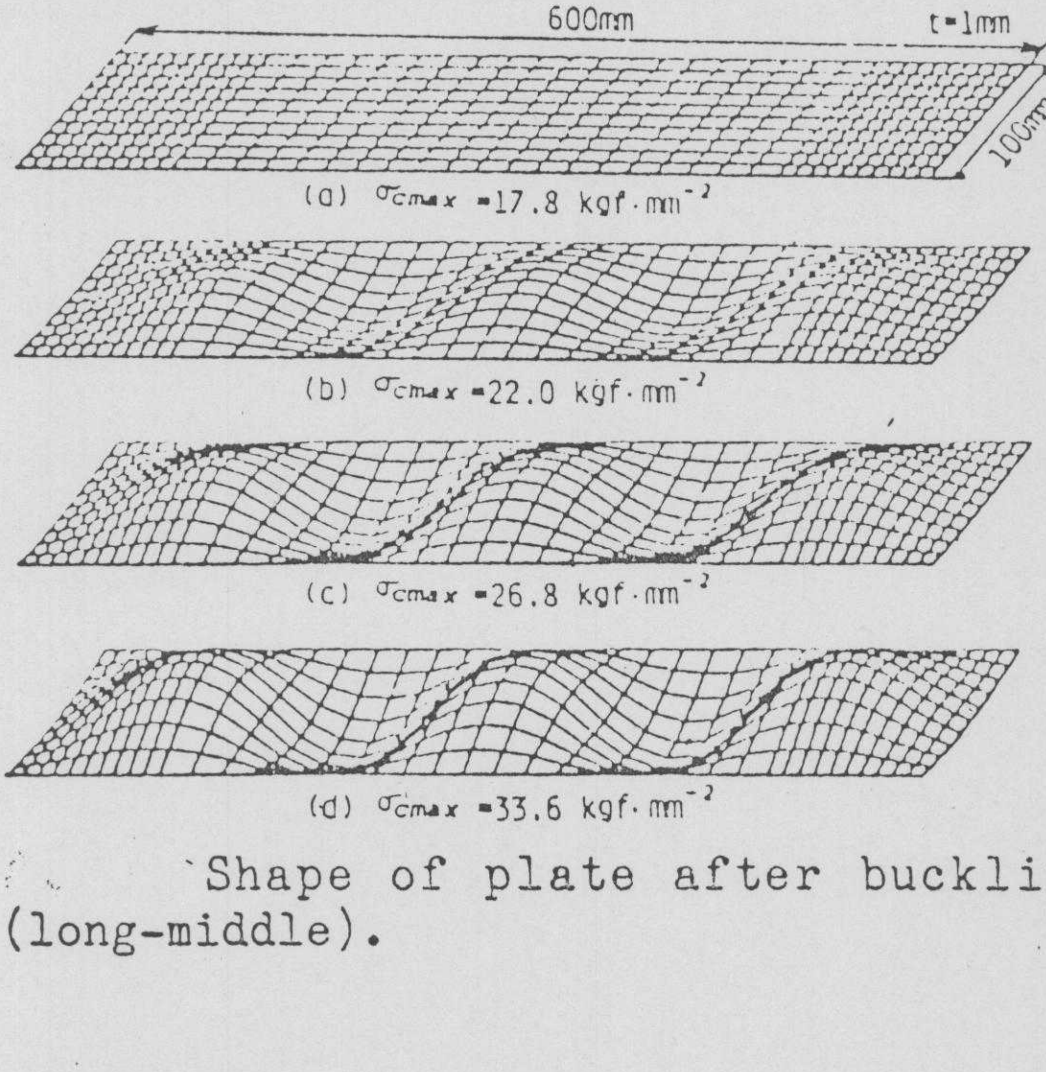
Calculated shape of strip during rolling.



Longitudinal residual stress during rolling.

N.Yukawa ,T.Ishikawa & Y.Tozawa , Numerical Analysis of the shape of Rolled strip,Proceedings Numiform '86 ,Gothenberg ,Publisher A.A.Balkema , Rotterdam 1986 ,p.249 - 254.

Fig.2(a) - Analysis of Sheet Rolling



Shape of plate after buckling (long-middle).

N.YUKAWA ,1986

Fig.2(b) - Analysis of Sheet Rolling

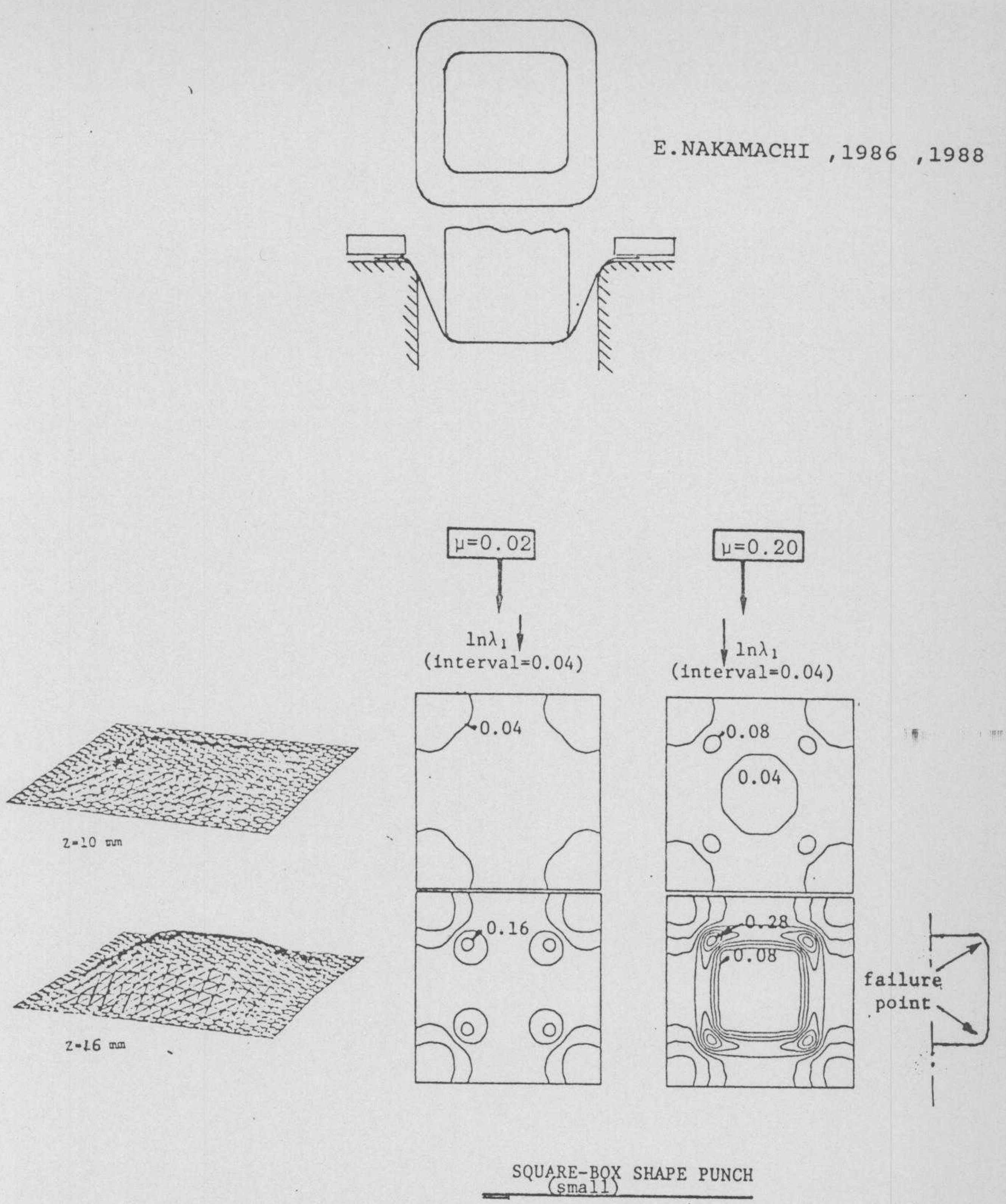
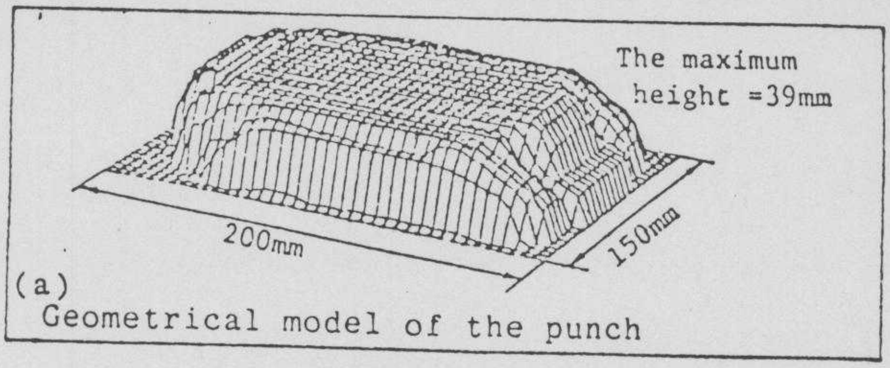
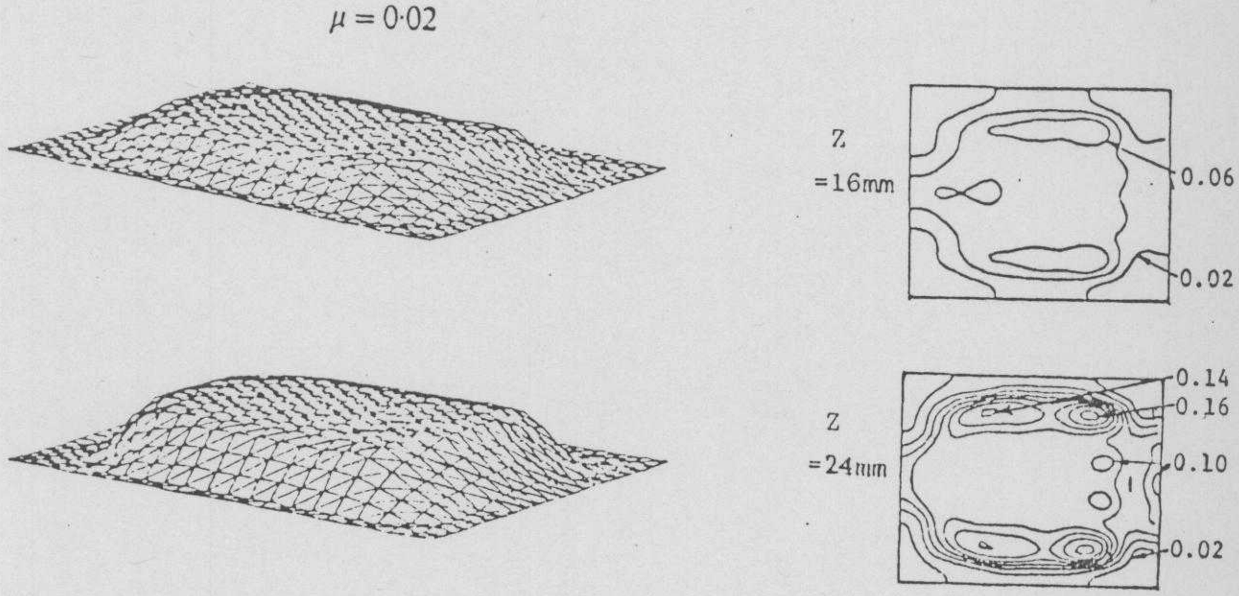


Fig.3(a) - Analysis of Stretch Forming of Square Punch

STRETCH FORMING



(a)
Geometrical model of the punch



Z= Punch travel

E.NAKAMACHI ,1986 ,1988

Eiji Nakamachi ,Finite Element Modelling of the punch press Forming of thin Elastic-Plastic Plates ,Proceedings Numiform '86 Gothenberg ,Publisher A.A.Balkema , Rotterdam 1986 ,p.333 -338.

Eiji Nakamachi , A Finite Element Simulation of the Sheet Metal Forming Process ,Int.J.for Num.Methods in Engg., vol.25 ,1988 p.283-292.

Fig.3(b) - Analysis of Stretch Forming of Oil-Pan

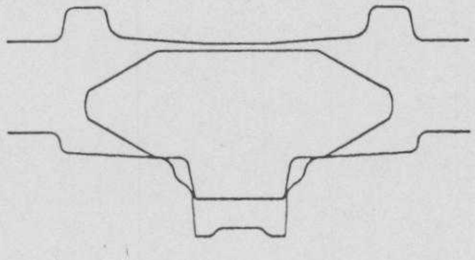


Figure a Dies and preform for a differential crown

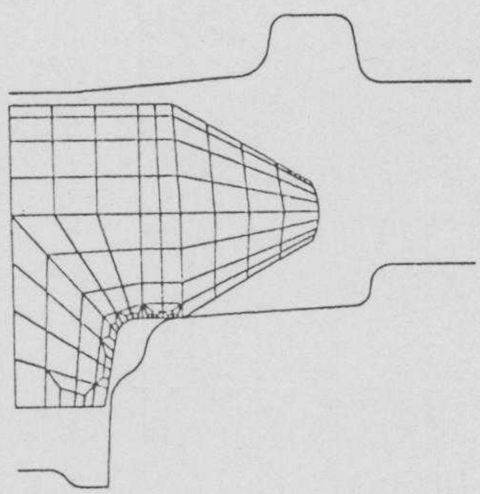


Figure b Initial mesh

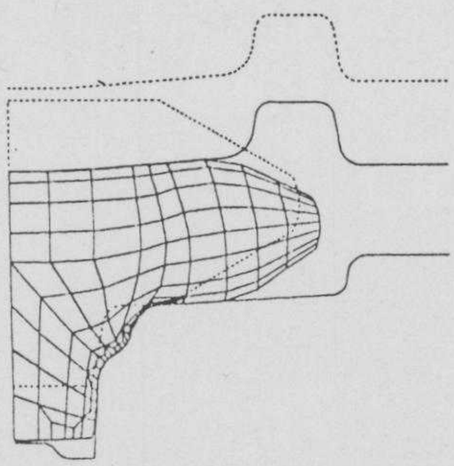


Figure c Deformed mesh after 20 increments

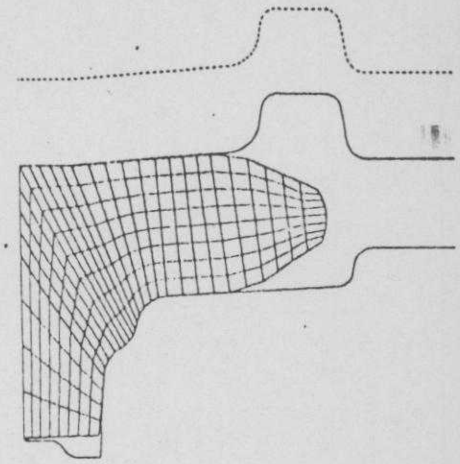


Figure d First rezoned mesh

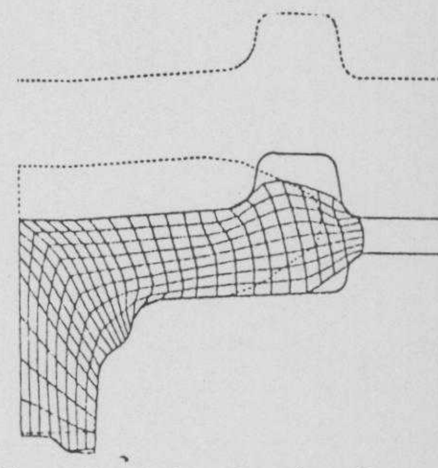


Figure e. Deformed mesh after 35 increments

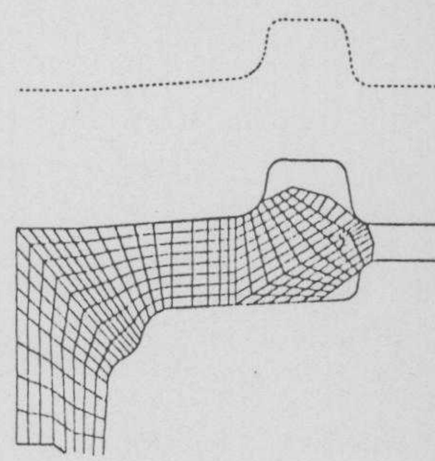


Figure f Second rezoned mesh

Fig.4 - Analysis of Drop Forging of Automobile Differential Crown

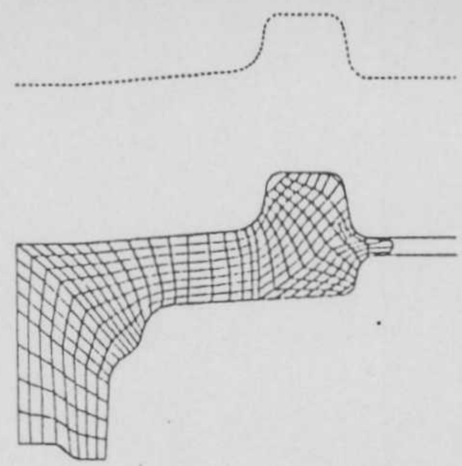


Figure g Deformed mesh after 46 increments

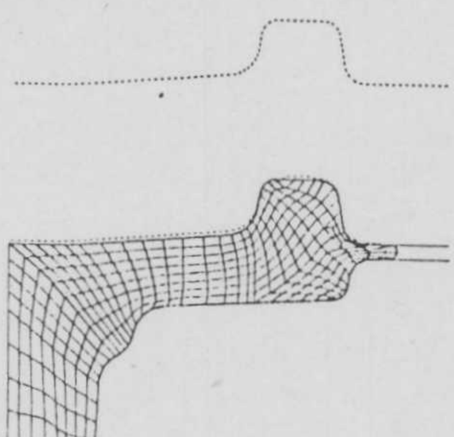


Figure h Deformed mesh after 51 increments

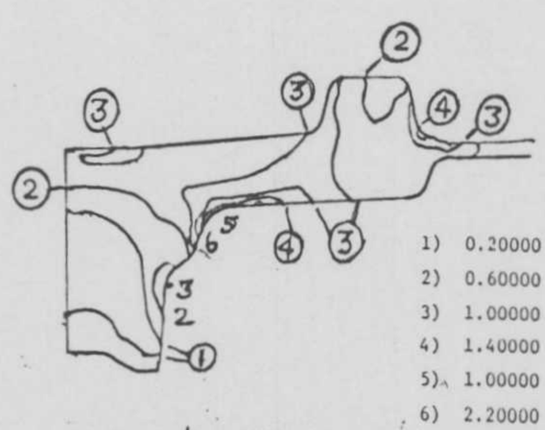
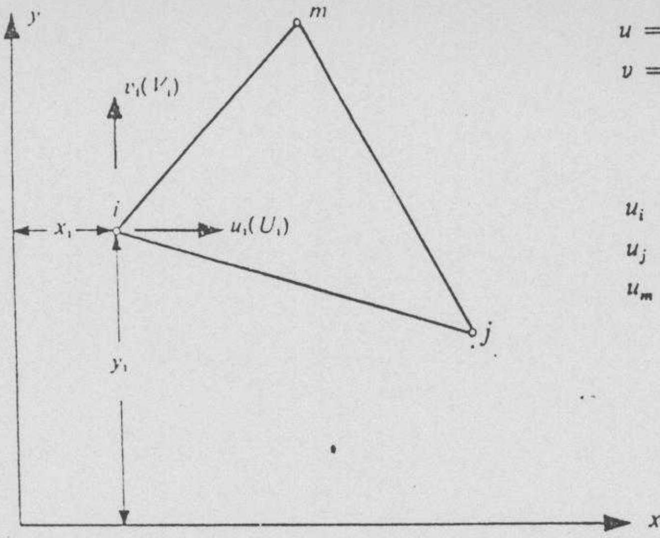


Figure k Effective strain contours after 51 increments

Fig.4 - (continued)



$$u = \alpha_1 + \alpha_2 x + \alpha_3 y,$$
$$v = \alpha_4 + \alpha_5 x + \alpha_6 y.$$

$$\begin{aligned} u_i &= \alpha_1 + \alpha_2 x_i + \alpha_3 y_i \\ u_j &= \alpha_1 + \alpha_2 x_j + \alpha_3 y_j \\ u_m &= \alpha_1 + \alpha_2 x_m + \alpha_3 y_m \end{aligned}$$

$$u = \frac{1}{2\Delta}\{(a_i + b_i x + c_i y)u_i + (a_j + b_j x + c_j y)u_j + (a_m + b_m x + c_m y)u_m\}$$

in which

$$\begin{aligned} a_i &= x_j y_m - x_m y_j \\ b_i &= y_j - y_m = y_{jm} \\ c_i &= x_m - x_j = x_{mj} \end{aligned}$$

$$2\Delta = \det \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_m & y_m \end{vmatrix} = 2 \quad \text{(area of triangle } ijm\text{)}.$$

$$\mathbf{u} = \begin{Bmatrix} u \\ v \end{Bmatrix} = [\mathbf{N}]\{\mathbf{a}^e\} = [\mathbf{I}N_i, \mathbf{I}N_j, \mathbf{I}N_m]\mathbf{a}^e$$

with $\mathbf{I}$ a two by two identity matrix, and

$$N_i = (a_i + b_i x + c_i y)/2\Delta \text{ etc.}$$

$$\mathbf{a}^e = \begin{Bmatrix} \mathbf{a}_i \\ \mathbf{a}_j \\ \mathbf{a}_m \end{Bmatrix} \qquad \mathbf{a}_i = \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}$$

Fig.5 An element of a continuum in plane stress or plane strain

$$\varepsilon = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x}, & 0 \\ 0, & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y}, & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} = [L]\{u\}$$

$$\varepsilon = [B]\{a^e\} = [B_i, B_j, B_m] \begin{Bmatrix} a_i \\ a_j \\ a_m \end{Bmatrix}$$

with a typical matrix B_i given by

$$B_i = LIN_i = \begin{bmatrix} \frac{\partial N_i}{\partial x}, & 0 \\ 0 & , \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y}, & \frac{\partial N_i}{\partial x} \end{bmatrix} = \frac{1}{2\Delta} \begin{bmatrix} b_i, & 0 \\ 0. & c_i \\ c_i, & b_i \end{bmatrix}$$

$$\begin{aligned} \varepsilon_x &= \sigma_x/E - \nu\sigma_y/E \\ \varepsilon_y &= -\nu\sigma_x/E + \sigma_y/E \\ \gamma_{xy} &= 2(1+\nu)\tau_{xy}/E \end{aligned}$$

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$$

$$\{\sigma\} = [D]\{\varepsilon\}$$

$$P.E. = \int_A \tfrac{1}{2}\{\epsilon\}^T\{\sigma\}\, t\, dA$$

$$[K]\{a\} - \{R\} = 0.$$

$$[K_{ij}] = \sum [k_{ij}]^e$$

$$[k]^e = \int [B]^T[D][B]\, d(\text{vol}),$$

General Form

$$[K]\{a\} + \{F\}_p + \{F\}_b + \{F\}_{\varepsilon_0} + \{F\}_{\sigma_0} - \{R\} = 0.$$

Fig.6