Operational Research in the Iron and Steel Industry

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THE name Operational Research was coined in the United Kingdom at about the time of the commencement of the Second World War. It was used initially to describe some work by civilian scientists in government service who were concerned primarily with the introduction of radar to the Royal Air Force to be used to warn of the approach of enemy aircraft. In the early days the efficiency with which defending fighter planes could be brought within range of the enemy was low in spite of the technical reliability of the radar equipment. Clearly what needed looking into and what the scientists proceeded to investigate was the whole warning system which included—besides the actual radar sets—the various control rooms where information from the radars was assessed and compared with other visual reports from Observer Groups and Police, and where decisions were made as to which fighters should be sent to intercept the enemy; the fighter aerodromes where the pilots had to be briefed on the position of the target and the time of take-off; and a complicated communications network connecting all these posts. They treated the investigation as a scientific problem. A logical picture was developed, the elements of the problem were isolated and hypotheses were set up on how these elements were connected and how variation in one element might affect another. Then the scientists went into the field and recorded how the various elements performed, and analysed these data to test how closely their logical model tallied with reality. The embryo model was then modified to explain satisfactorily the behaviour of the practical system they had observed. This process of logical description, observation and analysis is the common feature of all scientific method.

Factors impairing the efficiency of the warning system soon came to light. It was shown that under certain conditions the telephone communication network would inevitably become overloaded and that messages would be delayed; these delays would reduce the likelihood of the fighters intercepting the enemy. What is more, the scientists were able to indicate by how much the chance of interception would be reduced for each unit increase in delay in communication, and how the delay varied with the number of available comunication channels (number of telephone lines). Positive forecasts could thus be made of the effect on the efficiency of the system as a whole of specific changes in parts of it. The report was presented to the Air Staff and the suggested changes were made. As a result of this work and other subsequent Operational Research the effectiveness of the fighter defences which by the introduction of radar was increased to the point where one fighter could do the work of ten, was further doubled.

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The type of research named "Operational" then had two important characteristics: it entailed the application of the scientific method to the study of the whole complex within which an equipment was to be used rather than to the equipment in isolation; secondly, it meant an incursion by scientists into the province of management with the purpose of placing before management the best and most appropriate information upon which to base their decisions.

By the end of the war no arm of the United Kingdom or the United States Forces was without its Operational Research Group. And, not perhaps altogether surprisingly, when the war was over it occurred to many of the scientists who had taken part in Operational Research for the Forces, that a similar approach to the problems of industry might well produce results for the civilian economy commensurate with what had been done for the war effort. In the iron and steel industry on the initiative of Sir Charles Goodeve the Director of the British Iron and Steel Research Association an Operational Research Group of two men was formed in 1946. Today there are about 200 scientists working in recognised Operational Research Departments in the iron and steel industry; of these about 30 are in the BISRA Department.

It is impossible in the space available to do justice to all the work done by these departments; all that can be done is to give a few examples of work which typifies the methods used by Operational Research analysts in the industry.

Simple statistical analysis of works records

Managements are often confronted by the vexing fact that variations in product occur even when the plant is operated under apparently unvarying conditions; consequently planning production becomes extremely difficult especially when the plant is capable of making many different products and the order book only requires short runs of each. As an example, the case can be quoted of a firm which operated its rolling mill on a continuous basis and which wanted some sort of standard method to compare the performance of one shift with that of another. It was possible and usual for 65 different products to be produced in this mill, a product being defined as steel of a particular section and dimension. In the past the output of the shift had been measured in terms of tons rolled; this however was obviously unfair as the shift rolling the heavier sizes could get through many more tons than a shift committed to light sizes. Since wages were paid on the basis of tons rolled in a shift, the planning department had to keep making sure that no shift would be required to roll a preponderance of sizes which would adversely affect the wages paid or labour troubles would result. This inevitably means that rolling costs would not be at a minimum, as economic run sizes, number of roll changes and other considerations had to give way to the crucial consideration of ensuring an even wage.

The Operational Research analysts set about the problem by studying six months' production records and drawing up distributions of the tons rolled per hour for each product. A typical selection of these distributions is shown in Fig. 1, where for each product the horizontal scale is tons rolled per hour and the vertical scale is the number of cases in the sample when a particular rolling rate was achieved. Thus for sleepers the rolling rates varied from 25 tons per hour to 44 tons per hour; the average rate was 33 tons per hour. Examination of these distributions for each product revealed, for instance, as can be seen from Fig. 1, that there was no material difference between the average rolling rates for the different sizes of billet, 2 in., 2¼ in., 2½ in., 3 in., 4 in., etc. and that a single grouping would suffice. The indications of the distribution curves were checked statistically before the groupings were adopted, using the mathematical statistical techniques known as the Bartlett test of variance and Analysis of Variance.

It was finally found possible to sort the 65 products into 6 fairly homogeneous groups, the mean rolling rate of each group being typical of all the products in it. Next the group with the largest average rolling rate was denoted as the standard and all the other group average rolling rates expressed as a ratio of this standard. In Fig. 1, where the group at the bottom of the page was the standard, the ratios for all the other groups are the numbers down the left hand side of the page. In order to express it in terms of the standard rate the actual tons per hour achieved in practice for any product was multiplied by its corresponding ratio. For example an actual rolling rate of 40 tons per hour when making 75 lb F.B. rails is equivalent to a standard rate of $40 \times 1{\cdot}4$, that is 56 tons per hour.

With the number of groupings for standard rolling rates reduced to 6, it was possible to design a simple shift report which gave in tabular form the performance of each shift. For each hour of an eight-

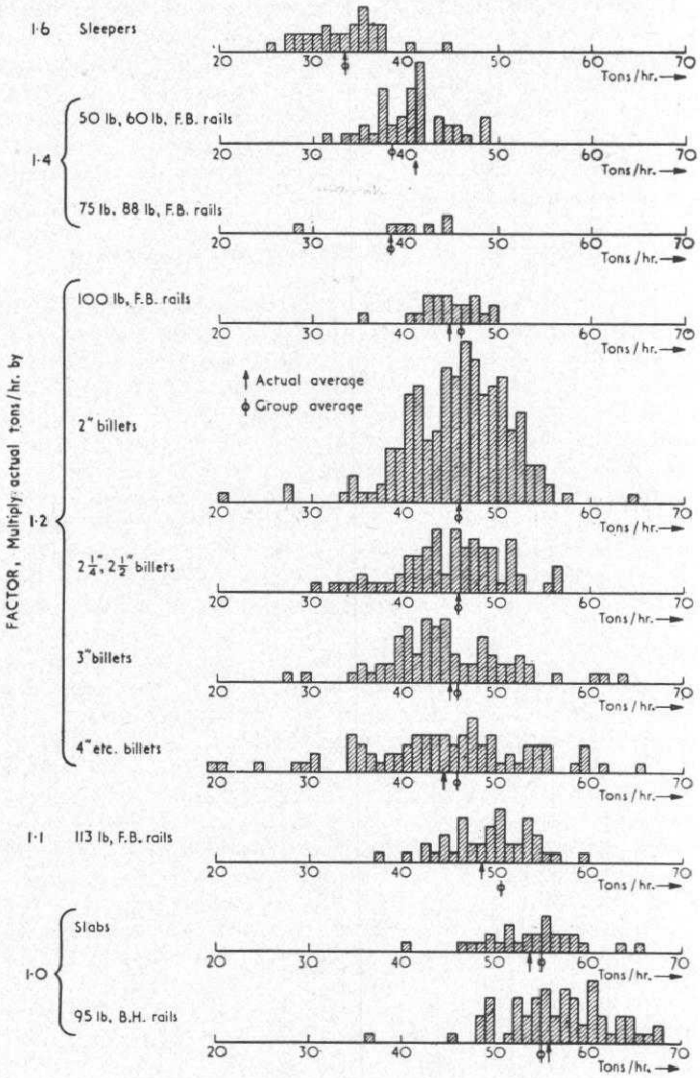


Fig. 1.

Net rolling rates, tons/hr, from works data.

hour shift, the weight of ingot, the number rolled, and (from a combination of the two) the weight rolled was recorded in columns. The weight rolled in the hour, multiplied by the factor appropriate to the particular group of product (e.g. 1·2 for billets), appeared in the next column. The total "corrected" weight rolled plus an allowance for stoppages (made on a pro rata basis) was recorded in the last column. The main advantage of this shift report was that it put all products on a comparable basis thus making it impossible for a foreman to use a stoppage or a change from a fast rolling to a slow rolling product as an excuse for low production. A special slide-rule was designed to be used with the shift report and no arithmetical calculations were required of the clerk in charge.

There is nothing very profound about the methods used in this first example. Nevertheless, by their means an effective tool was devised for eliminating variations in rolling rates and multiplicity of products as obstacles to the best utilisation of resources. A very high proportion of Operational Research work in fact requires technical methods no more complicated than these.

An application of the theory of queues

Some years ago the British steel industry in considering its future development plans was searching for a policy concerning the importing of foreign ore. The buying and importation of foreign ore was and still is the responsibility of a central agency acting on behalf of the whole industry and the ore is resold to the individual companies as it comes over the ships' side at the receiving port. In order to facilitate decisions BISRA was asked to investigate the problems involved in importing iron ore; the Operational Research Department was set the task of investigating the factors which affect the cost.

The cost of the ore to the companies can be considered as the sum of the price of the ore once it is in the holds of the ship at the foreign port, the cost of transporting the ore in the ship, and the cost of unloading the ore at the receiving port. The price of the ore loaded on a ship at the foreign port depends upon market values and many other factors, most of which the British industry cannot directly control except by the normal processes of commercial bargaining and negotiating contracts on the open market. It was decided therefore to concentrate on the other two aspects of the cost, the transportation by ship and the unloading.

In considering this problem it was apparent that each ship used to transport ore can be considered to have the following work cycle:

(*i*) Waiting and loading at the foreign port taking an average time t_l.

(*ii*) Travelling from foreign port to home port and back, taking average time t_v each way.

(*iii*) Waiting outside the home port for a vacant berth taking an average time t_w.

(*iv*) Being unloaded and revictualled etc. at the home port taking average time t_u.

The time (T) for each ship to work this complete cycle can be expressed by the equation:

$$T = t_l + 2t_v + t_w + t_u \qquad \ldots \qquad \ldots \quad (1)$$

Since the time a ship waits and takes to load at the foreign port must be considered as outside the control of the British industry alone it was, for the purpose of the study, considered to be a constant for a given size and type of ship. The value which this constant assumed in practice was ascertained from the records of ships' voyages.

The travelling time $2t_v$ was also considered to be a constant for a given size and type of ship travelling between two particular ports. This step had its justification in the fact that in general ships were always operated at what the owner considered to be the most economical speed. For the same reasons average voyage times for particular ships and between particular ports were also obtained by a study of the records of ships' voyages.

The time a ship may have to wait, or queue, for an empty berth at the unloading port (t_w) will depend upon how many other ships need to use the same berths, how many berths are equipped for unloading iron ore, and how fast a ship can be unloaded once it is at the berth.

The last part of equation (1), the time the ship is occupied with unloading (t_u), depends on the type and size of ship, the unloading facilities on the berth, and the type of ore which has to be unloaded.

Obviously, the shorter the time (T) denoting the complete work cycle for each ship, the more voyages each ship can perform in a given time and therefore the smaller will be the total number of ships of a given type and size which are necessary to import a given amount of ore; and there will be a corresponding saving in transport charges per ton. Such a reduction in the number of ships importing a given quantity of ore could be effected by reducing the average wait for a berth (t_w) and/or the time to unload the ship (t_u).

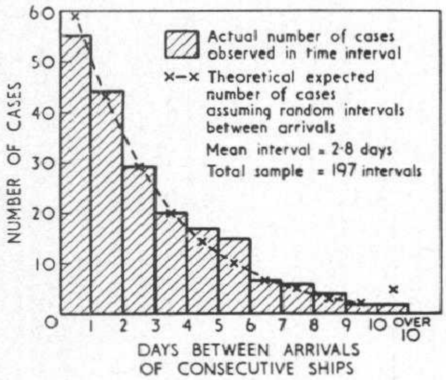
The necessity of waiting for a berth would, in theory, be avoided if ships' journeys could be scheduled so as to make a ship arrive just as the previous ship at the berth had finished unloading. In practice variations in the time to unload a given ship which depend, for example, upon the type and condition of the ore it carries, combine with storms and delays at sea and the foreign port to upset prearranged schedules. Despite considerable scheduling of journeys there are inevitably times when two ships arrive at the same berth more or less together and one of them has to wait; there are other times when the interval between one arrival and the next is so long that the berth remains empty and the unloading equipment and its operators idle.

Such fluctuations in the demand on the port cannot be eliminated. On the other hand a faster turnround of individual ships at the berth will reduce the average time ships have to wait for a berth, that is both t_w and t_u would be shorter. A speed-up in turnround at the berth cannot of course be achieved at no additional cost. Faster equipment costs more to buy and instal than slower equipment, and may cost more to operate. Also, by clearing the berth more quickly, there will be more occasions when there is no ship in the queue waiting to occupy the berth and equipment and labour are idle.

The cost of installing faster equipment, however, may exceed any saving achieved by reducing the number of ships required by shortening the time they have to queue. As there will always be some queuing no matter how fast the unloading equipment installed, the best position will have been achieved when the sum of the costs due to ships (including queuing, etc.) and the costs due to unloading are as small as possible, that is at a minimum.

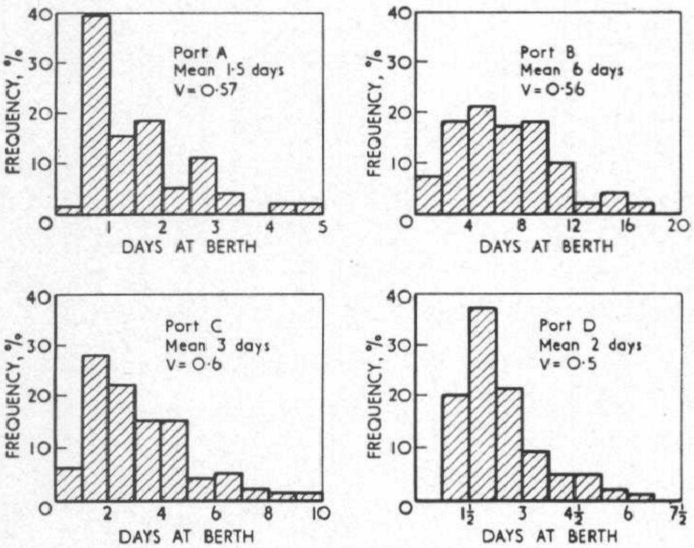
To predict the achievement of this position it was necessary to discover in the first place how the speed of unloading at a berth affects the average time a ship must wait in the queue. Records of many ports and ships were examined to arrive at the time elapsing between successive arrivals of ships to be unloaded. A typical distribution of such time intervals for a particular port is shown in Fig. 2 (*a*).

At this port data were obtained on 197 consecutive ship arrivals: the shaded histogram indicates that in 55 cases there was an interval of between 0 and



(a)

ARRIVALS OF SHIPS AT TYPICAL PORT



(b)

Fig. 2.
Time to turnround ship at berth.

1 day, and in **44** cases an interval between 1 and 2 days and so on. The figure also shows (dotted line) the proportions of ship arrivals which would be expected if these arrivals were truly random. This theoretical distribution can be obtained from the following equation:

The proportion of intervals that are at least t days in length but less than $(t+1)$ day is

$$e^{-\lambda t}\left(1-e^{-\lambda}\right) \quad \ldots \quad \ldots \quad (2)$$

where λ is the mean rate of arrivals 0·36 ship per day.

It can be seen that the theoretical points obtained from equation (2) are a very reasonale approximation to the observed events. Similar correspondence between the theoretical and actual distributions was found at all ports. It will be seen later that this finding was an important and convenient one.

Data were also obtained from the port records on how long ships took to be unloaded. For these figures to become meaningful they had to be classified according to ship size and type and examined in the light of the type of unloading equipment available at the berth. In Fig. 2(*b*) are examples of how the turnround time of particular types of ship at the berth varied at each of four different ports. The average time a ship was at the berth can be seen to vary considerably from one port to another due to the different unloading facilities available; the equally considerable variations within each port are due to such things as crane breakdowns, and different types and conditions of ore. Nevertheless, one important consistency exists between the various ports: the variation compared with the mean, expressed by the coefficient of variation is for practical purposes the same in each case.

The findings from the above data were now used to establish the relationship existing between unloading times and the average time a ship would have to wait for a berth. Here a theory originally developed by the Danish Telephone Service was used; their problem was to decide what was the relationship between the length of time a telephone user would have to wait for the exchange to answer and the facilities available at the telephone exchange for answering his call. The mathematics of the theory rests on the assumption that the time interval between any two calls from subscribers is random and that the mean and the coefficient of variation of the distribution of times which the operator takes to deal with calls are known. The theory is equally applicable to the transportation-unloading system for ore ships. For the intervals between telephone calls substitute the intervals between ship arrivals; for the time the operator takes to deal with individual calls, substitute the time to turnround the ship at a berth. The data, too, are in the right form to fit the theory. The observed distribution of arrivals of ships at a port approximates closely to a random distribution as has been shown and is illustrated by Fig. 2 (*a*); the mean time to unload a ship can be calculated from a knowledge of the capabilities of the unloading equipment and the size of ship; the coefficient of variation is known to be a constant substantially independent of the mean as is demonstrated in Fig. 2 (*b*). The average delay per ship (t_W) can thus be calculated from the following equation (assuming that there is only one berth suitable for unloading iron ore in the port).

$$t_W = \frac{\lambda t_u^2(1+v^2)}{2(1-\lambda t_u)} \qquad \ldots \qquad \ldots \quad (3)$$

where as before λ is the mean rate of arrivals
t_u is the average time a ship is at the unloading berth and v is the coefficient of variation of the times at the berth.

If there is more than one berth at the port a more complicated expression has to be employed.

With this mathematics and a knowledge of the capabilities of unloading equipment, the size of ships and the average arrival rate of these ships at a port, it was clearly possible to make a fairly accurate forecast of the average time each ship will be delayed in the queue waiting for a berth. This knowledge collated with values of the cost of ships when at sea and when in port and the cost of unloading equipment capable of particular unloading rates, made it possible to build up a picture of the probable cost of the whole operation. The results for a port expected to handle one million tons of ore per annum brought in ships of 8,000 tons capacity are shown in Fig. 3.

The total cost per ton of ore unloaded when various rates of unloading are provided at the berth is here

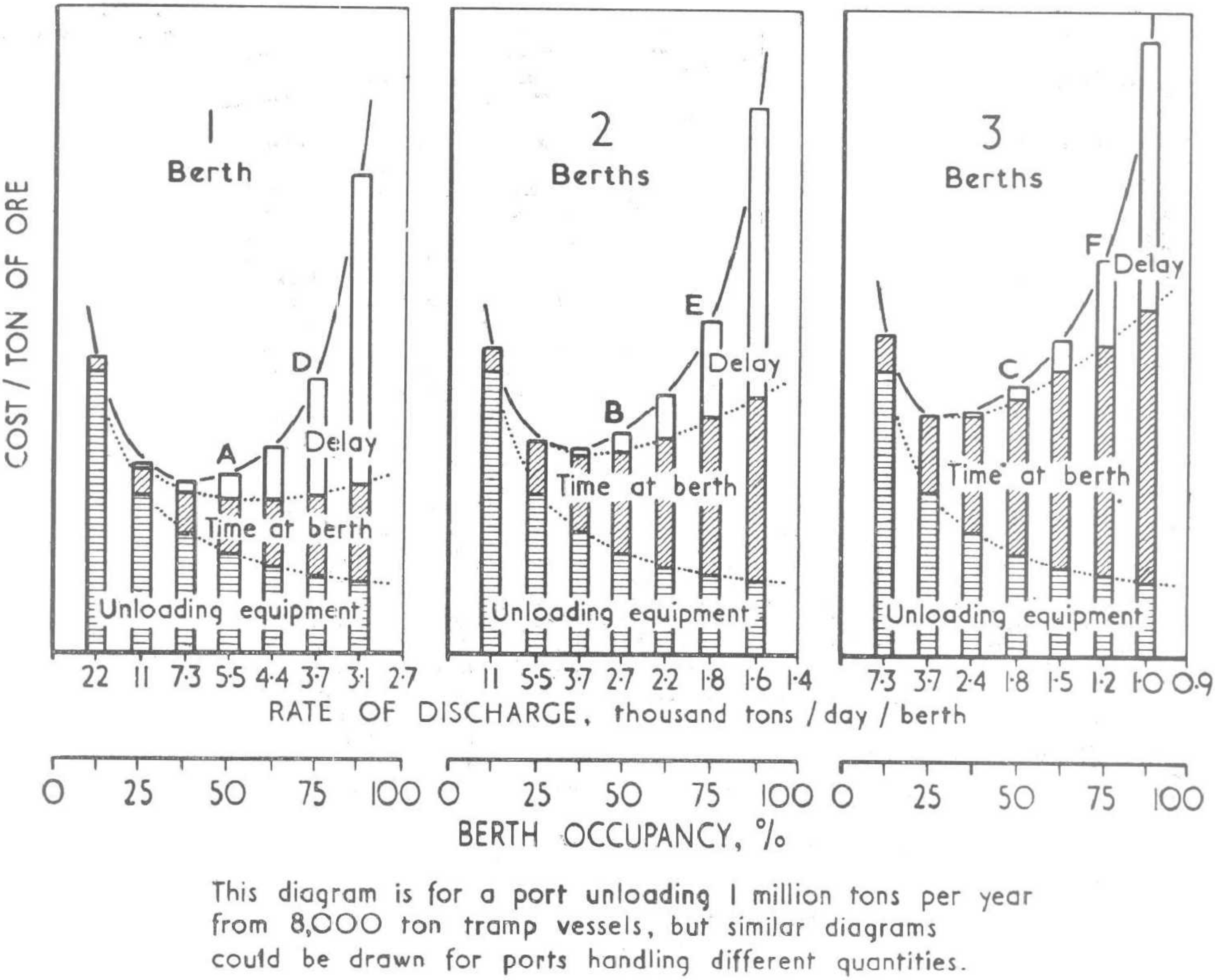


Fig. 3.
The total cost of discharge.

divided into its component parts of cost of ship delays in the queue, cost of ships time whilst at the berth, and cost of providing and operating the unloading equipment ; incidentally it was found during the investigation that although the latter varied with the rate of discharge of which it was capable, it was sensibly independent of the various types of equipment available. It can be seen from Fig. 3 that it is cheapest to provide only one berth with a high capacity unloading plant at a port handling this amount of ore rather than two or three berths with slower equipment. Also it is interesting to note that the cheapest situation arises when the berth is designed to be occupied (the berth-occupancy) for only half its time on average. The latter is important as it is contrary to the ingrained habit of the port operator who intuitively aims at the highest possible utilisation of his equipment.

Fig. 3 summarises the results of the Operational Research study. This, however, is not quite the end of the story ; in practice the ship is owned and operated by one man and the port by another. The question arises how to persuade both to operate in a manner which does not necessarily appear to either of them as the most efficient way of running his own business, but which is nevertheless the optimum way of operating the whole system. The link between the two was finally achieved by paying the port operator a bonus for quick turnround of ships which made it worth his while to instal the right amount of equipment for operating the whole system in the most economical way. The introduction of this bonus made the situation, which appeared optimal from the port operator's isolated point of view, synonymous with the situation which optimised the whole system.

The project is typical of operational research in that it looks at the whole system to discover the interdependencies, attempts to design a model of the system, in this case a mathematical model, proves the adequacy of the model with actual operating figures and uses the model to forecast the effects of any changes in facilities and operating procedure. The study also shows how scientific techniques derived for one set of circumstances can be vicariously applied to an apparently quite different problem.

It will be appreciated that there are numerous operating problems in the steel industry which involve queues of one kind or another, for example the reception sidings at a steelworks, all stocks of raw materials, of interprocess materials, and of finished products, the size of the engineering maintenance force and many others. A considerable volume of mathematical theory specifically applicable to these various problems has been especially developed in the last few years.

The simulation of operations

There are many problems in steelworks where a mathematical description of the situation becomes extremely complicated ; in order to solve the mathematical equations simplifying assumptions may have to be made which are so drastic that the answers, though theoretically sound, bear very little relation to what would happen in the practical situation which the equations were intended to represent. The following example typifies such a situation and explains how nevertheless the problem may be successfully tackled.

The problem arose in a company which operated a steel melting shop with an open hearth furnace, and two smaller electric furnaces ; it was the custom to work the open hearth furnace continuously, but to work the electric furnaces for only a proportion of the available shifts. The company planned to increase their steel production by working all the furnaces in the shop on a continuous basis, but were doubtful whether a possible inadequacy in the facilities for getting the raw materials to the furnaces would not produce delays. They wanted a forecast of delays in order to decide on the advisability of improving or increasing the existing handling facilities.

The essentials of the steel making process can be described, in this instance, by considering the flow of materials through the shop. The raw material was mainly steel scrap and pig iron delivered by road and rail, and put to ground in the scrap bay adjacent to and parallel to the main line of the melting shop (see Fig. 4). The scrap and pig were handled by two overhead cranes (A and B) fitted with magnets. These cranes also loaded the scrap into pans when it was required by the furnaces. The magnets on the cranes were then replaced by slings, and the scrap pans were picked up two at a time, weighed, and set down at one of the points of access to the melting shop. They were then collected by a crane on the melting shop stage which set them down on a gantry until the furnaces were ready. In order to charge the furnaces, the pans were picked up one at a time by the charging machine, and the contents emptied into the bath of the furnace. Other materials were also charged, but the quantities involved were so small that their effect on congestion and thus delays to charging was negligible. Empty pans were put back on to the gantry, and eventually returned to the scrap bay by the stage crane and one of the scrap bay cranes.

The bulk of the scrap was such that generally the furnaces could not be charged to capacity in one operation ; a period of time had therefore to be allowed for the first charge to be melted before more scrap could be charged. The steel was then melted and refined, and was eventually tapped into a ladle on the side of the furnace remote from the scrap bay ; the ladle was held by the single overhead crane in the casting bay.

After a furnace had been tapped, fettling of the hearth was required, and the furnace was then prepared for the next charge. Periodically the refractories making up the furnace lining had to be

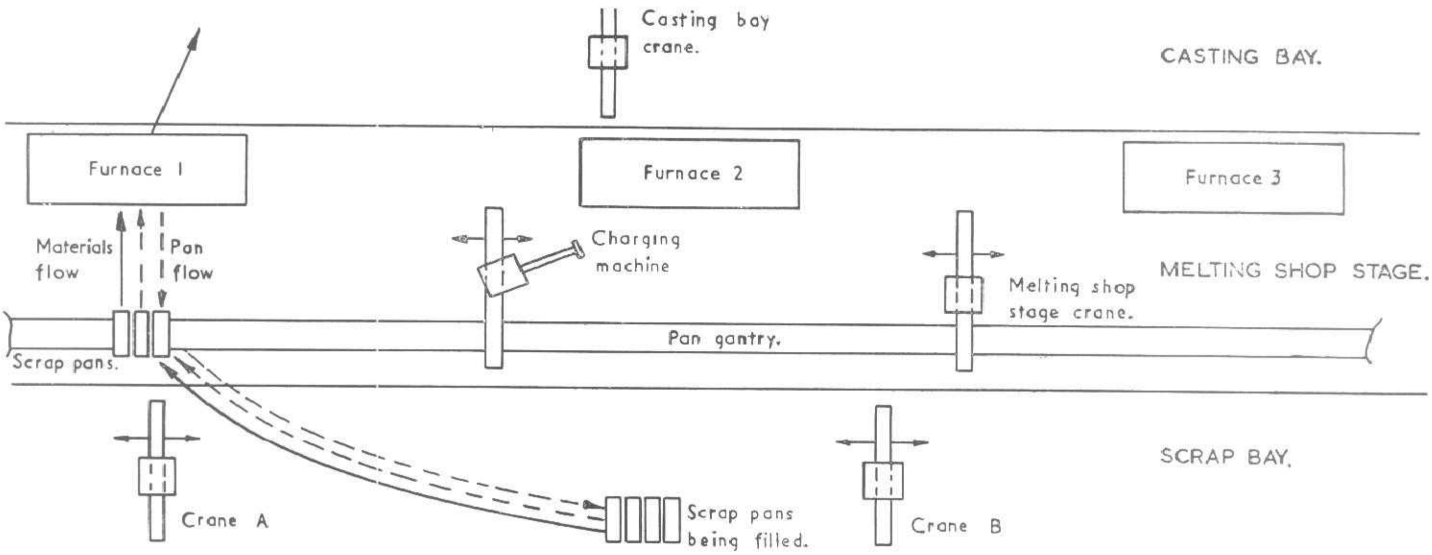


Fig. 4.
Schematic diagram of the melting shop.

replaced and this represented the main cause of furnace downtime.

In this particular shop, the single charging machine and the stage crane ran on the same track and therefore both could not be operated at the same time.

From first principles it was clear that during a furnace charging period two main types of delay might arise which could be characterised as follows :

(*a*) "Pan delays", when there were no full pans or not enough full pans available on the stage when a furnace was ready to be charged.

(*b*) "Bunching delays", when more than one furnace required charging at a given time.

Other delays might arise outside a charging period, for example, when more than one furnace was ready for tapping at a given time, or when a piece of equipment broke down.

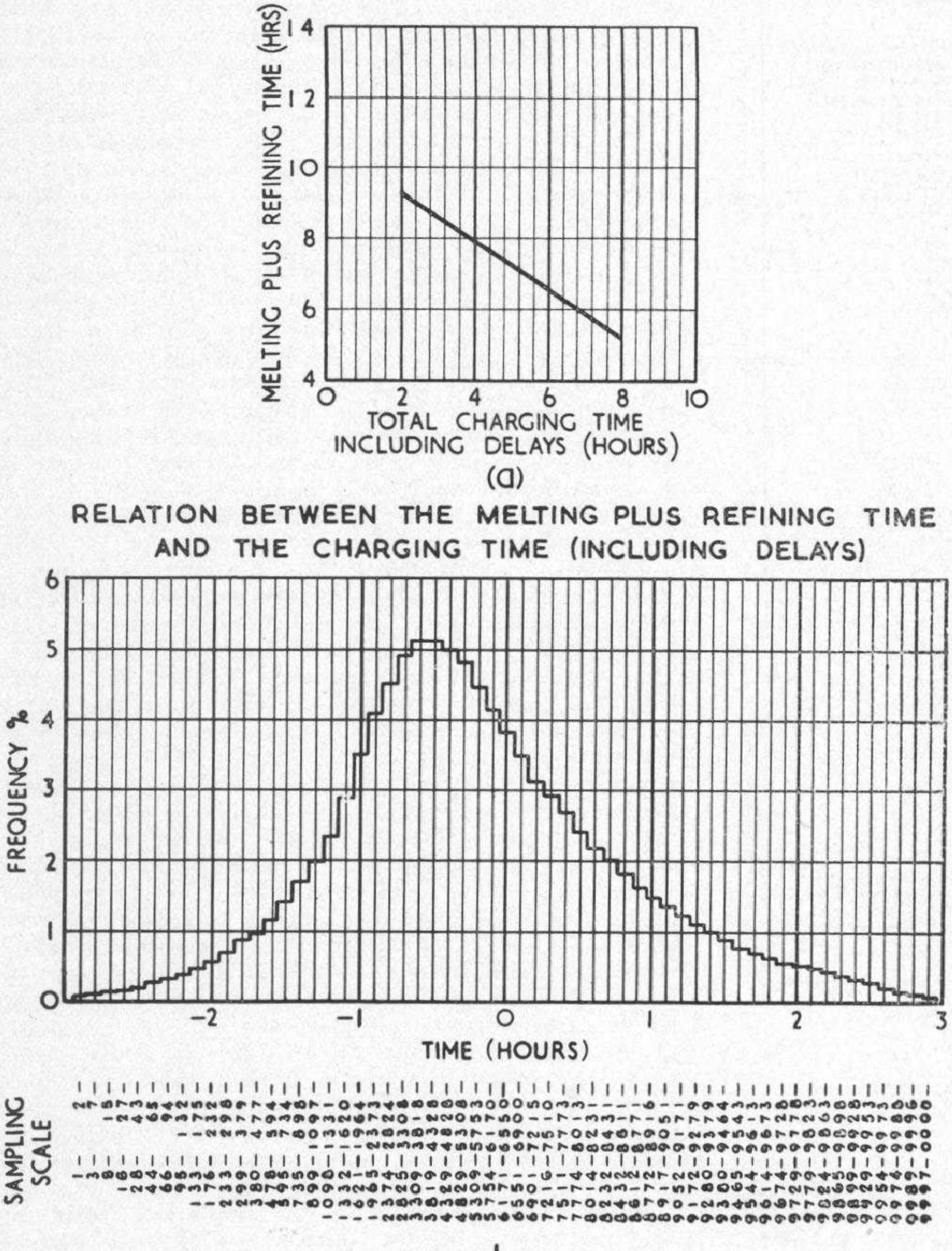
A preliminary study was made of the existing practice in the melting shop using time studies and works records for the previous year. From these records it appeared that delays had been relatively small ; it could be argued that short delays might not be recorded and that an estimate depending on the records might under-estimate their importance. However the bonus scheme was such that a pay allowance was made for time lost due to delays and there was therefore an incentive to record all delays.

Before attempting to estimate the effect of the existing delays during charging it was important to realise that there was a restriction on any saving that could be made in the furnace tap-to-tap times by eliminating these delays. Some melting of the scrap (and pig) must occur during the time charging was actually taking place and a reduction in the charging time thus implies that part of this melting would have been lost. The actual association between charging time and the subsequent time required to complete melting-plus-refining is shown in Fig. 5 for one of the furnaces. Similar associations were discovered for the other two furnaces.

It can be seen that as charging time decreases, subsequent melting time increases ; for example, a saving in charging time of one hour is associated, on average, with an increase of 36 minutes in melting-plus-refining time, which is equivalent to saving only 24 minutes in the tap-to-tap time. This relationship is obviously very important when estimating the effect of charging delays both with existing practice and with the proposed increase in production.

The straight line relationship shown in Fig. 5 was obtained by a statistical analysis of data obtained from works records ; the technique is known as regression analysis and includes an estimate of the risk associated with accepting the assumption that it is in fact a straight line relationship. In this case the risk was found to be very small. Of course results in practice will vary somewhat about this line and the lower diagram of Fig. 5 is a representation of the limits within which a practical value is likely to lie about the average obtained from the regression line in the top diagram. For example for a charging time of 4 hours on average the melting-plus-refining time would be about 8 hours, but any individual case might lie within plus or minus 3 hours of this value i.e. between 5 and 11 hours.

From such figures it was possible to estimate by how much on average the production could be increased if it were possible to eliminate all delays to charging. This gave an upper limit to production with the present rate of working ; and a similar estimate could be made for what would happen if all the furnaces were worked full time. To make the



(a)

RELATION BETWEEN THE MELTING PLUS REFINING TIME AND THE CHARGING TIME (INCLUDING DELAYS)

(b)

DISTRIBUTION OF CORRECTIONS TO MELTING PLUS REFINING TIME FOR A GIVEN CHARGING TIME

Fig. 5.

Production times for Furnace 1.

latter a practical estimate however an estimate was required of what delays could actually be expected with continuous production and how much these would reduce the throughput.

The steelmaking process is inherently variable; for example, the range of tap-to-tap times for Furnace 1 was 9½ hours to 23 hours (see Fig. 6) and hence any attempt to forecast the future production would be unrealistic if it were simply based upon average furnace times.

The effects of congestion (interference), which are an indirect consequence of this variability would be ignored. A method of analysis must therefore be used which takes account of these moment-by-moment variations.

For this reason the different production systems considered had to be worked through graphically (i.e. the operations of the shop *simulated* on paper step by step as realistically as possible) to provide more accurate estimates of the output that could be expected. The coincidental occurrence of the adverse factors which cause congestion, and hence delays, will thus arise in much the same way as they would in practice. The basic ideas of this "simultation" technique are briefly explained below.

If sufficient data were available, a past week's operation of the melting shop could obviously be represented on a chart showing the state of each furnace, the operation of each piece of ancillary equipment (including its breakdown time) and the whereabouts of full and empty scrap pans, each recorded against a continuous time scale. Similarly it is possible to prepare a similar chart for what might be called a representative week. In this case instead of using a particular observed time for a particular day, a choice would be made from a complete range of times obtained from works records. The resulting simulation of a week's production probably would not be exactly the same as any actually observed but it would be typical, in the sense that it certainly would represent a week which might arise sometime.

In practice, the simulation of such an operation is by no means straightforward. First, the logic of the operation must be appreciated; for example, the sequence in which operations occur must be known, in addition to the reasons why certain actions must take place at certain times. Secondly, possible associations between different production factors must be evaluated (e.g. the relationship

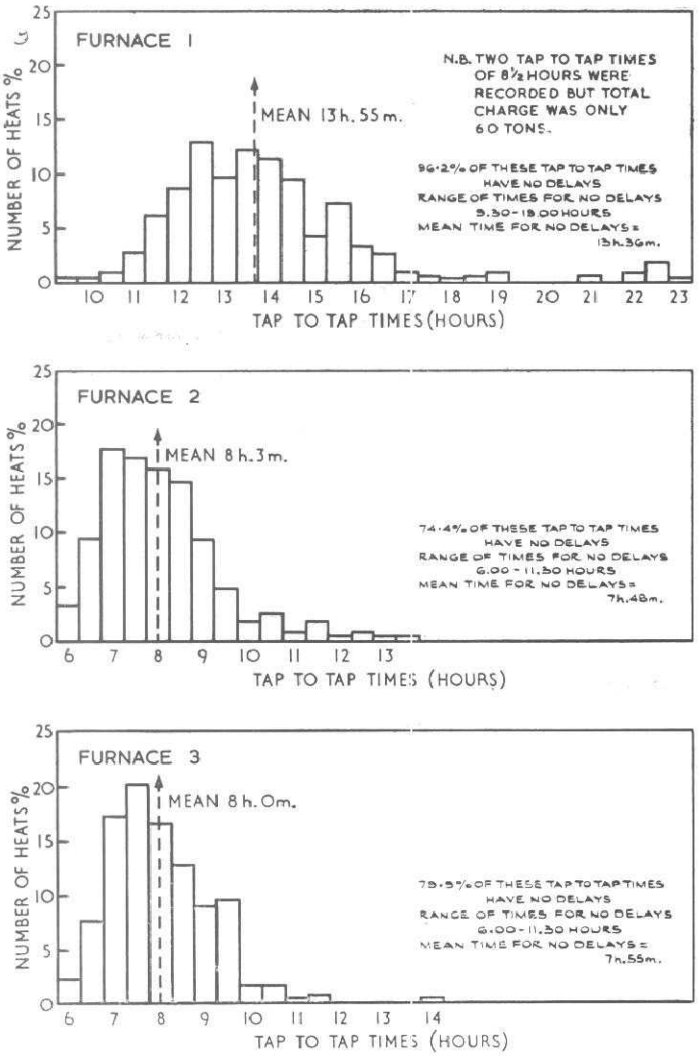


Fig. 6.

Distributions of tap to tap times (including recorded delays).

between charging time and the subsequent melting-plus-refining time, and the relationship between the number of pans charged and the charging time). These associations may not be obvious and the advice of the technologists on the spot can be helpful. Finally, to simulate a particular operation the selection of the data must be made without bias from the mass available.

This is done in the following way, taking as an example the choice of a melting-plus-refining time for a particular heat. Suppose it is already known that charging of the furnace has taken 4 hours, then from the top diagram in Fig. 5 it can be seen that an average melting-plus-refining time of 8 hours could be expected ; however this is only an average time and for a particular case this figure will not do as it stands. The lower diagram of Fig. 5 represents the variation of actually observed figures about this average. This diagram has had associated with it a sampling scale extending from 1 to 10,000 divided so that the numbers 1 to 2 are associated with the value minus 2·9 hours on the diagram, and the numbers 3 to 7 are associated with the value—2·8 hours and so on. The proportion of the 10,000 numbers associated with each value on the time scale is directly related to the frequency represented by the vertical scale on the diagram with which that value has been observed to arise in practice. In order to choose a figure from this diagram for a particular case one of the numbers 1 to 10,000 is drawn so that the chance of drawing any one of them is equally likely. Supposing in this case the number drawn were 400, this number is associated with minus 1·7 hours on the diagram and therefore the melting-plus-refining time to be used in this case, with a charging time of 4 hours, would be (8–1·7) hours, that is 6·3 hours. The next time a melting-plus-refining time is required for a furnace a similar process is gone through necessitating a new choice from the lower diagram.

To produce a diagram of the operations of the shop, of which Fig. 7 is an example, distributions of the times of all the basic activities are required. The operations of each piece of equipment in the shop are indicated on the diagram on a horizontal line calibrated in units of time synonymous with the time in the life of the shop. Each equipment is allocated a separate line across the diagram, and the equipment appropriate to each line is noted down the left hand side. Thus the activities of the shop can be represented on the diagram in the sequence in which they would have to take place in practice. To obtain a value to enter for the duration of each particular event a separate random choice is made from the appropriate distribution of times. Competing demands for equipment will occur for example where two furnaces require the charger at the same time, one or other furnace must wait ; these and other delays are marked in on the diagram as they develop. It would be noted that these delays are actually generated by the process and not themselves chosen from distributions of past events.

In working through such a simulation rules of operation have to be devised so that a correct choice can be made when, for example, alternative duties arise at the same time for the same piece of equipment. These rules of operation are the things which control the way in which the shop will operate. Alternative sets of rules can be tried in different simulations to assess which are likely to provide the best operation of the shop.

In the particular project under discussion simulations of the future operation of the shop not only provided estimates of future production but also indicated to management the best way of organising and operating the shop. These rules of operation which were in a way a by-product of the study were in fact regarded by management as invaluable.

The technique of simulation is a most powerful one in Operational Research. One of its drawbacks is that the mechanics of carrying it out can become cumbersome and tedious, particularly as the operations must be simulated over considerable periods to produce reliable answers. Fortunately this drawback is cancelled out by modern electronic digital computers which can be programmed to work through them automatically and which moreover can do so much more accurately than is possible by hand.

An application of mathematical analysis

There are many steel works in the United Kingdom where a wide variety of products can be made, each from a range of raw materials and each on alternative plant. Such situations for example exist in that part of the industry concerned with the cold rolling of steel strip. Many complications arise in preparing production programmes for strip departments. In the first place, hot rolled strip, which is the raw material of the cold rolling process, is usually available in a range of sizes and in general more than one size can be used to make a given product. Again, the plant available will often allow several alternatives in the sequence of operations by which the finished product can be manufactured from any one of the suitable sizes of hot rolled strip.

The nature of the problems arising can be appreciated by considering the following imaginary example. Table I lists the possible ways in which a given product could be produced, using either of two suitable sizes of hot rolled strip, and plant consisting of two rolling mills and one slitting machine of specified operating characteristics. (It is assumed that no processes other than rolling and slitting are needed). Even when hypothetically certain alternatives are excluded as obviously more costly than others in the list, there remain eight ways of making the product from one size of strip and four of making it from the other. For this simple example it may well be that, considering the handling costs, machine running costs, etc., there would be an obviously best method of producing any product in the sense that by this route the overall conversion cost would be least.

If a complex of plant is available for the manufacture of a number of products, it is not necessarily true that the best overall production programme is one that ensures the manufacture of each product by its most economical route. Possible routes, cost, machine occupancy and profit margins allowed by the pricing system in force are all factors which must be considered in deciding the optimum programme. In these circumstances the selection of a production programme which will yield the maximum profit (or achieve any other optimal condition desired) can be a complex procedure.

This situation has been examined for a works where the processes involved in production of cold rolled strip were pickling, scale breaking, rolling, grinding and annealing; in the manufacture of any product some, or all, of these processes were required.

A mathematical description of the processes can be illustrated by considering the demand on one piece of plant in meeting a given production programme. Suppose that in one week it is desired to make the following:

T_1 tons of product 1
T_2 tons of product 2
... ...
... ...
... ...
... ...
T_n tons of product n

where n is the total number of products.

The capacity available on one of the rolling mills is R hours per week, and the capacity on that mill to roll 1 ton of product 1 is R_1 hours, and for 2 tons of product 1 is $R_1 \times 2$ hours and for T_1 tons of product 1 is $R_1\ T_1$ hours; in other words it is supposed that there is a linear relationship between total machine capacity required and tonnage rolled. Such an assumption is rarely true in the strict sense, but in practice the error introduced by accepting it is usually small.

When all the n products are considered the total machine capacity required must be the sum of the individual requirements $R_1\ T_1 + R_2\ T_2 + R_3\ T_3 + \dots\dots\dots + R_n\ T_n$ and for feasibility this total must be less than, or equal to, available capacity R of the machine; symbolically:

$$\sum_{j=1}^{j=n} R_j\ T_j \leqslant R \qquad \dots \qquad \dots \quad (4)$$

(A particular product may not require capacity on this particular machine; in this case the appropriate R_1, R_2, R_3 etc. will be zero).

Equations of the type of (4) can be derived for each piece of plant in the works. These represent the simplest possible mathematical description of the operation of the works and are enough to ensure that the programme envisaged will not overload the plant capacity. However in order to achieve a more realistic representation of the situation, further requirements may have to be met. For example only a limited demand may exist for some products; for others a long term contract already negotiated may demand a certain minimum production each week.

Constraints of this kind may be handled by constructing additional equations. For example, if the maximum amount of product 1 which can be marketed per week is D_1 and none is to be warehoused, then an equation of the following type is required ensuring that T_1 is not greater than the upper limit:

$$T_1 \leqslant D_1 \qquad \dots \qquad \dots \quad (5)$$

On the other hand, if a contract has stipulated a delivery of at least D_2 tons of product 2 per week then an equation of the following type would be applicable:

$$T_2 \geqslant D_2 \qquad \dots \qquad \dots \quad (6)$$

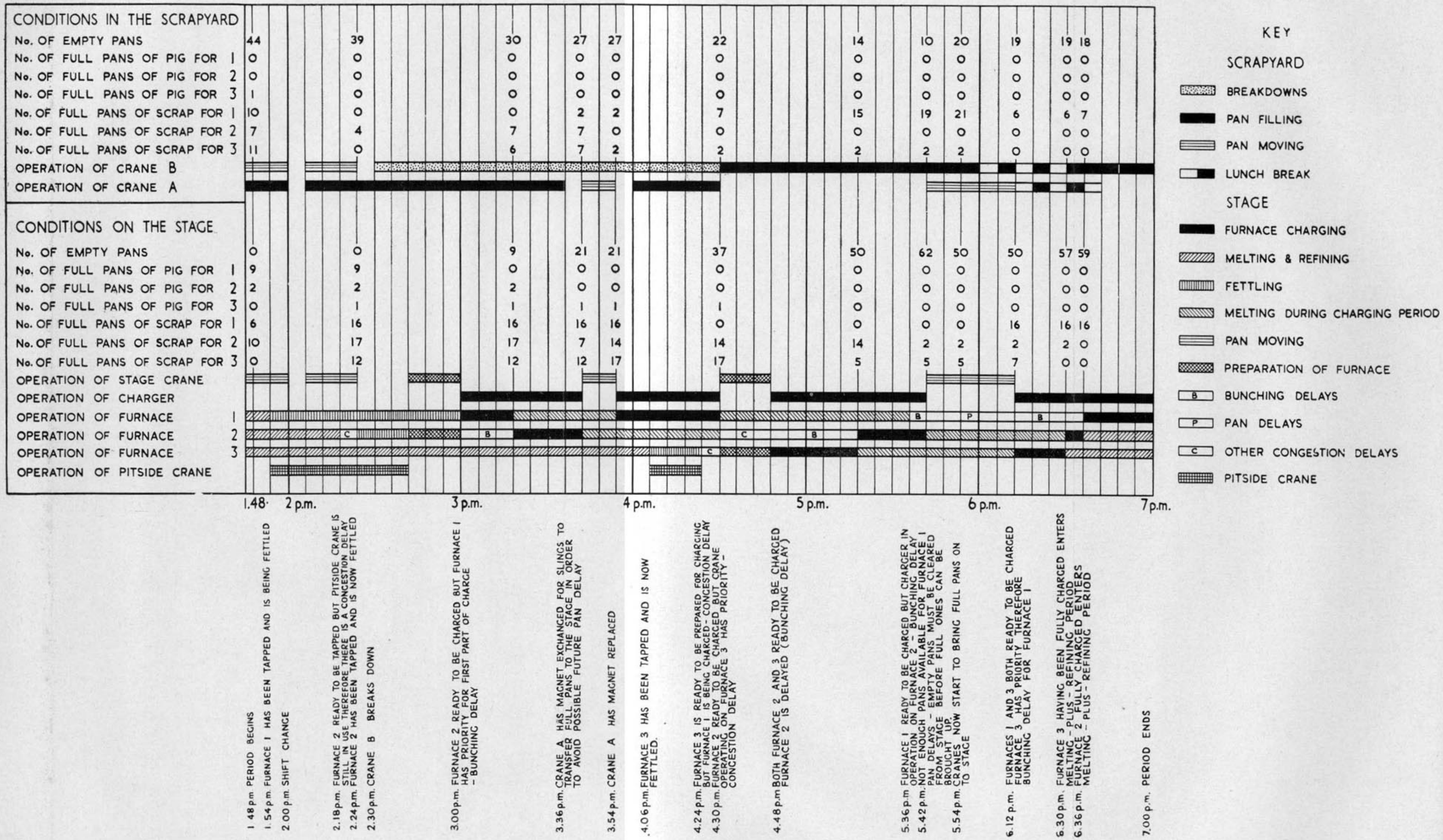


Fig. 7.
A representative period in a simulation.

Table I

Descriptive representation of problem of programming production

Plant available... Rolling Mill M_1 : Max. Width 4b : Max. Thickness 4a, Min. Thickness 2a, Time per Unit Length t_1
M_2 : ,, ,, 2b, ,, ,, 4a, ,, ,, a, ,, ,, ,, ,, t_2
Slitter S : ,, ,, 4b, ,, ,, 3a, ,, ,, a, ,, ,, ,, ,, t_3
Assume **1** pass required for reduction of "a" irrespective of width.

Hot rolled strip : (a) Length (L) per ton c Width (W) 4b Thickness (T) 4a
(b) ,, ,, ,, ,, 8/3 c ,, ,, 2b ,, ,, 3a

Product required : 1 ton of Width b, Thickness a.

	Hot-rolled strip L	W	T	M_1 L	W	T	Time	M_2 L	W	T	Time	S L	W	T	Time
A	c	4b	4a	(1) 4/3c	4b	3a	ct_1	(4) 8c	2b	a	$4ct_2$	(3) 4c	2b	2a	$2ct_3$
				(2) 2c	4b	2a	$4/3ct_1$					(5) 16c	b	a	$8ct_3$
B	c	4b	4a	(1) 4/3c	4b	3a	ct_1	(4) 16c	b	a	$8ct_2$	(3) 8c	b	2a	$2ct_3$
				(2) 2c	4b	2a	$4/3ct_1$								
C	c	4b	4a	(1) 4/3c	4b	3a	ct_1	(4) 8c	2b	a	$4ct_2$	(2) 8/3c	2b	3a	4/3c
				(3) 4c	2b	2a	$8/3ct_1$					(5) 16c	b	a	$8ct_3$
D	c	4b	4a	(1) 4/3c	4b	3a	ct_1	(5) 16c	b	a	$8ct_2$	(2) 8/3c	2b	3a	$4/3ct_3$
				(3) 4c	2b	2a	$8/3ct_1$					(4) 8c	b	2a	$4ct_3$
E	c	4b	4a	(1) 4/3c	4b	3a	ct_1	(3) 4c	2b	2a	$8/3ct_2$	(2) 8/3c	2b	3a	$4/3ct_3$
								(4) 8c	2b	a	$4ct_2$	(5) 16c	b	a	$8ct_3$
F	c	4b	4a	(1) 4/3c	4b	3a	ct_1	(3) 4c	2b	2a	$8/3ct_2$	(2) 8/3c	2b	3a	$4/3ct_3$
								(5) 16c	b	a	$8ct_2$	(4) 8c	b	a	$4ct_3$
G	c	4b	4a	(1) 4/3c	4b	3a	ct_1	(4) 16c	b	a	$8ct_2$	(2) 16/3c	b	3a	$4/3ct_3$
				(3) 8c	b	2a	$16/3ct_1$								
H	c	4b	4a	(1) 4/3c	4b	3a	ct_1	(3) 8c	b	2a	$16/3ct_2$	(2) 16/3c	b	3a	$4/3ct_3$
								(4) 16c	b	a	$8ct_2$				
I	8/3c	2b	3a	(1) 4c	2b	2a	$8/3ct_1$	(2) 8c	2b	a	$4ct_2$	(3) 16c	b	a	$8ct_2$
J	8/3c	2b	3a	(1) 4c	2b	2a	$8/3ct_1$	(3) 216c	b	a	$8ct_2$	(2) 8c	b	2a	$4ct_3$
K	8/3c	2b	3a					(1) 4c	2b	2a	$8/3ct_2$	(3) 16c	b	a	$8ct_3$
								(2) 8c	2b	a	$4ct_2$				
L	8/3c	2b	3a					(1) 4c	2b	2a	$8/3ct_2$	(2) 8c	b	2a	$4ct_3$
								(3) 16c	b	a	$8ct_2$				

Dimensions are those *after* carrying out of operation.

Bracketed numbers indicate order in which operations are performed.

Slitting to unequal widths not considered.

Consecutive slitting operations not considered.

No scrap losses.

Rolling time proportional to length.

Slitting time proportional to length before slitting, not that in Table.

Total time required on each machine for each sequence of operations.

Sequence M/C	A	B	C	D	E	F	G	H	I	J	K	L
M_1	$7/3ct_1$	$7/3ct_1$	$11/3ct_1$	$11/3ct_1$	$1ct_1$	$1ct_1$	$19/3ct_1$	$1ct_1$	$8/3ct_1$	$8/3ct_1$	0	0
M_2	$4ct_2$	$8ct_2$	$4ct_2$	$8ct_2$	$20/3ct_2$	$32/3ct_2$	$8ct_2$	$40/3ct_2$	$4ct_2$	$8ct_2$	$20/3ct_2$	$32/3ct_2$
Slitter	$10ct_3$	$2ct_3$	$28/3ct_3$	$16/3ct_3$	$28/3ct_3$	$16/3ct_3$	$4/3ct_3$	$4/3ct_3$	$8ct_3$	$4ct_3$	$8ct_3$	$4ct_3$

When a set of constraining equations of the type (4), (5) and (6) have been prepared to represent the practical situation as accurately as is required, it is possible to find many sets of values of T_1, T_2, T_3 ....T_n which would satisfy all the equations simultaneously. The question then arises as to which is the best set of values to use. A criterion is required to decide what is meant by best; several such criteria are possible, depending on circumstances. One might be that plant should be utilised to the greatest possible extent, another that the greatest tonnage of product should be made, another that the total profit per week should be a maximum.

Consider the last named of these. Profit can be considered as the difference between selling price and the sum of raw materials, etc., costs and conversion costs. A profit P in these terms can be calculated for each 1 ton of product; thus the profit on T_1 tons of product 1 will be $T_1\,P_1$. The total profit derived from a production of T_1, T_2, T_3.........T_n tons of products 1, 2, 3......n, will therefore be the sum of the individual profits on each product $T_1\,P_1$, $T_2\,P_2$, $T_3\,P_3$......$T_n\,P_n$; this can be expressed symbolically as follows:

$$\text{Total profit} = \sum_{j=1}^{j=n} T_j\,P_j \qquad \ldots \qquad \ldots \quad (7)$$

Now for each production programme obtained by solving all the simultaneous equations of the type (4), (5) and (6) above there will be a set of values for T_1, T_2, T_3......T_n, which can be substituted in expression (7) in order to assess the profit applicable to each particular programme. In considering how to solve these equations simultaneously it is immediately obvious that trial and error would be possible but if there are many possible products and machines this is impracticable.

Fortunately a systematic method of choosing the best set of values is available in a numerical analysis technique known as *linear programming*. It is still an iterative process in that successive values of the variables have to be inserted in the equations and effect on profit assessed each time. However the method ensures that with each change in the values of the variables a smaller profit cannot result, and it also gives a positive indication when a stage is reached where any further change in the values cannot yield a greater profit. The computation though frequently still formidable is in many cases well within the capabilities of modern electronic digital computers.

At a particular company where the task was to assess what products should be made to achieve the maximum possible profit, there were still 3,078 variables to be evaluated even when obviously less profitable alternatives were excluded, and only 13 machines were involved. To solve the equations works records had to be examined to obtain process times of each possible product on each appropriate machine, and to test that the assumption of a linear relationship between tonnage of product and the time required on the machine was valid. The term product was interpreted to mean strip of stated characteristics produced from a specified raw material by a specified set of operations.

It was apparent that, for all practical purposes, unlimited capacity could be assumed to be available for pickling, scale breaking, and annealing. Consequently, these operations affected only the estimation of production costs which were required in order to calculate the profit coefficients for each product. The limiting factors in the system were the capacities available on the various rolling mills and on the grinding and slitting machines used in producing the type of strip in question.

From the Cost Office breakdown of running costs for each of the machines, the figures relating to direct operations were extracted and an hourly rate calculated, assuming full working capacity. (If the optimum solution indicated unused capacity on any machine, its rate could be reassessed on the basis of capacity actually utilised and the calculations repeated using the revised data.) With these machine costs and production times known, production costs for each of the products were calculated and combined with the price lists for hot and cold-rolled strip to give a set of profit margins for each ton of product. Out of the total profit the company would have to pay all fixed overheads which were not taken into account in the calculations; but these would not change with the level or type of production.

The linear programme carried out on a computer indicated that with the ideal programme of production the gross profit per week could be five times larger than the present running of the plant on the normal programme. However, this calculated programme took no account of possible restrictions such as the lack of immediate market for some products in the quantity in which they would be produced. This omission was deliberate as by subsequently recalculating the programme with such a restriction included, the answer could be used to show the management the effect of this market restriction in terms of lost potential profit. In other words it could be used to give management a guide on how much was worth spending on advertising or other sales campaigns to increase the market for particular products. Analysis of this kind is usually known as assessing the "opportunity cost".

It must be re-emphasised in conclusion that linear programming though valuable in solving many complex problems has one big drawback in that the relationships between the variables concerned must approximate to linearity.

Organising for operational research

The examples of Operational Research studies quoted do not cover all the methods that have been used or all the types of problem to which it has been applied in the steel industry in the United Kingdom and the United States of America. However they do indicate its basic conception of being prepared to utilise any and all of the established methods of science for the purpose of investigating the general operating and planning problems of industry.

It is interesting to note the origins of the various methods mentioned in the above examples. Mathematical statistical analysis was first derived to explain the results of biological and agricultural experiments. Queuing theory was developed by electrical engineers for the design of telephone systems. Simulation, or "Monte Carlo" experiments, as they are sometimes called, were first used by physicists to help in the exploration of the atomic nucleus. Linear programming was developed in economics to help in the description of a country's economy. This borrowing of methods from pure and applied science is the crux of Operational Research; hardly any method can strictly be claimed to have been derived purely for operational research.

That the personnel of an Operational Research Department should be drawn from many different sciences is thus hardly surprising. For example in the BISRA department there are engineers, physicists, chemists, metallurgists, economists, mathematicians, statisticians, psychologists and logicians. The only thing that these people have in common is that they have all been trained to use the scientific method. On the other hand a knowledge of the technology of iron and steel making is not by any means a necessary prerequisite for conducting Operational Research in the steel industry; it might be nearer the mark to suggest that too intimate a knowledge of the ingrained practices and prejudices, which come almost inevitably with production experience in an industry are a definite handicap since they tend to make it difficult to see the wood for the trees. The advantage of being on the outside looking in is very great in this particular type of work. Naturally a complete ignorance of the industry has its dangers and it is essential that Operational Research staff should be able to obtain the confidence and cooperation of the technologists and the management of the industry. Thus staff must be chosen not only for their general scientific abilities, but for those qualities which will make them acceptable to the people they are intended to help and advise in the industry.

In carrying out a project it is usual to employ a team of analysts so that the disciplines of several different sciences can be brought to bear on the problem. This multi-discipline team is thought by many to be an essential feature of the Operational Research approach. Thus in starting an Operational Research Group it is unwise to appoint fewer than three people. General experience appears to show that it is usual to include in this team someone trained in statistics, someone who is good at mathematics, and a good all round engineer or practically minded pure scientist such as a physicist. As the team expands many groups tend to include an industrial psychologist for it must be remembered that in all problems the activities of human beings are involved.

Finally it should be said that, because Operational Research demands very good staff, it is never cheap; for this reason it should be used only on vital problems which are not already being adequately covered by other departments, and also that results should be reported to whoever can take action, whether he be at the top of the management structure or on the shop floor. On his part the Operational Research analyst has to remember that his studies are useless if he cannot convince the management of the validity of his analysis and of the potential and practicability of his results.

Conclusion

This paper outlines the approach to problems which is typical of Operational Research. In the space available it was possible to describe only a few of the ways in which it has been applied in the iron and steel industry of the United Kingdom. In its task of assisting management in decision making Operational Research provides forecasts of the overall effects of alternative policies by employing scientific methods to describe the interplay of men, materials and machines which together comprise the industry, and by investigating the various ways in which control may be exercised. The specific methods used have been shown to range from straightforward studies of the problems of organisation and operation within a department, to more complicated mathematical and statistical techniques which are used to explain the effects observed at works, and to assess the likely results of changes in procedure.

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