MODELLING OF MULTIPLE TRANSVERSE CRACKS IN $[\pm\theta /90^0]_s$ ANGLE - PLY LAMINATES

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Abstract

Multiple transverse cracking is one of the most common failure modes in composite laminates. In this paper, transverse crack behaviour in $[\pm\theta/90^0]_s$ angle-ply composite laminates under uniaxial loading has been studied by 3D finite element method. Energy method is used for calculating strain energy release rate and stiffness reduction as the crack extends in the width direction. A control volume having a single transverse crack has been used for the analysis. Eight noded isoparametric layered composite elements with three degrees of freedom per node have been used to discretise the control volume. Load is applied in longitudinal direction via uniform applied displacements through the $\pm\theta$ outer layers. Strain energy release rate and stiffness reduction are calculated for different crack lengths, crack densities and fibre orientation angle θ of the outer layers. From the analysis it has been observed that strain energy release rate increases with the increase in fibre orientation angle θ of the outer layers. However, the variation of strain energy release rate in width direction is negligible. Stiffness reduction has been noticed with increase in fibre orientation angle θ and crack density.

Introduction

Applications of composite materials are increasing due to inability of conventional materials to meet high strength/weight and high stiffness/weight ratios. Because of the heterogeneous construction, failure mechanisms of the composite materials are much more complicated than that of conventional isotropic materials. Multiple transverse cracking is one among the most common modes of failures in composites. Generally, transverse cracks initiated in 90° ply are extended in the fibre directions from the free edges. Although the formation of transverse cracks does not lead to catastrophic failure in pressure vessels and liquid containers, their presence may lead to leakage and decrease the residual strength. In pressure vessels multiple transverse cracking can also cause significant stiffness reduction.

Composite laminates with transverse cracks have been studied by many investigators using different methods. These methods can be classified into two groups - (i) the Shearlag analysis [1,2,3,4] and (ii) the Variational method [5,6].

Laws and Dvorak [1] showed the loss of stiffness in composite laminates as a function of crack density and crack density as a function of applied loading. Lim and Hong [2] have studied the effect of the 90° layer thickness and constraining effect of the 0° layer on the transverse cracking behaviour of cross-ply laminates. Stress distribution and stiffness ratio was given by Lee and Daniel [3] using the simple shearlag method assuming that the next set of cracks develop when the maximum axial stresses in those plies reach the transverse strength of the layer. Zhang et al.[4] studied the effect of matrix cracking on the inplane stiffness properties of $[\pm\theta_m/90_n]_s$ composite laminates loaded in tension. Stiffness reduction due to transverse cracks in cross-ply laminates was studied by Hashin [5], Varana and Berglund [6] using variational method on the basis of principle of minimum complementary energy. The main assumption in this approach is that normal ply stress in the load direction is constant over the ply thickness. Gulid et al.[7] used 3D finite element method to study the variations of fracture parameters as a transverse crack grows across the width of the laminate. The dependence of crack behaviour on the 90° layer thickness has been studied by some authors [8,9] using ply discount theory. They found that the crack spacing decreases with the increasing applied stress and increases with the increasing ply thickness. In this paper, an attempt has been made to study the effect of fibre orientation angle θ of the outer layers on the crack behaviour in width direction using 3D finite element method. Stiffness ratio and strain energy release rate are calculated for different crack lengths as well as for different crack spacings. Closed form analytical solutions are not available in literature for varied crack lengths as the crack propagates in the width direction and hence the results are compared only with the full width crack problems.

Analytical Study

Process of Crack Multiplication

Let us condider a composite laminate with two transverse cracks in the 90° layer. Garret and Bailey [8] have found that the longitudinal stress is maximum at the mid point between two cracks and it was assumed that the new crack initiates at the centre of the two original cracks. The new crack propagates as the load increases and gradually spans the entire width, and with further increase in the load, two new cracks develop in between these three cracks and this process is repeated with the increase in load. The distance between the cracks decreases, as the number of cracks are increased.

Strain Energy Release Rate

Control volume with a single transverse crack of length 'a' as shown in Fig. 1 has been used for the analysis. Control volume consists of two outside ±θ layers of equal thicknesses 'b' on either side of the laminate and the central 90° layer of thickness 2d and subjected to uniform uniaxial displacements applied through the ±θ layers. The elastic energy stored in the laminate is given by

$$U = \frac{1}{2}\left(\sum R\right)v \quad (1)$$

where, U is the total elastic energy in the laminate

$$\sum R = \text{Resultant reactive forces due to applied displacements}$$

v = Displacement

Eqn. (1) can be used to calculate the change of strain energy with incremental change in crack length as

$$\frac{dU}{da} = \frac{\left(U_{a+da} - U_a\right)}{da} \quad (2)$$

where,

U_a = Strain energy with crack length 'a'
U_{a+da} = Strain energy with crack length "a + da"
da = Incremental crack length in width direction.

The strain energy release rate 'G' is given as

$$G = \frac{1}{2d}\frac{dU}{da} \text{ for uniform applied displacement} \quad (3)$$

where

2d = Thickness of 90° ply

Stiffness Reduction

Stress-strain relation for the uncracked laminate is given by

$$\sigma_o = E_o \varepsilon_o \quad (4)$$

where,

σ_o = Average axial stress of the uncracked laminate

ε_o = Applied strain

E_o = Young's modulus for the uncracked laminate

Stress-strain relation for the cracked laminate is given by

$$\sigma_c = E_c \varepsilon_o \qquad (5)$$

where,

E_c = effective axial stiffness of the cracked laminate and

σ_c = Average axial stress of the cracked laminate.

Strain energy U_0 of the uncracked laminate is given by

$$U_0 = \frac{1}{2}\sigma_o \varepsilon_o V \qquad (6)$$

From Equations (5) and (6)

$$U_0 = \frac{1}{2}\varepsilon_o^2 E_o V \qquad (7)$$

where,

V = Volume = 4WbS
W = Width of the laminate
b = Thickness of each ±θ laminate
2S = Crack spacing

Similarly, strain energy U_c of the cracked laminate is written as

$$U_c = \frac{1}{2}\varepsilon_c^2 E_c V \qquad (8)$$

From Eqns (7) and (8) the normalised stiffness ratio, (a measure of stiffness loss due to crack propagation) is evaluated for uniform applied displacement through the ±θ layers as

$$\frac{E_c}{E_o} = \frac{U_c}{U_o} \qquad (9)$$

Eqn (9) can be used to calculate the rate of stiffness loss as the crack propagates towards full width.

Finite Element Modelling

A 3D finite element analysis has been carried out by the finite element code ANSYS5.0 (Swasnson Analysis Inc system). A symmetric half section of the control volume due to mid laminate symmetry about x-y plane (Fig. 2) is considered. Eight noded isoparametric composite elements with three degrees of freedom per node and varying fibre angle capabilities have been used for the descretization. Each layer was modelled to have one element in thickness direction and twenty elements in width direction. The crack surfaces 'MNQP' have been modelled by configuring the two sets of nodes in the plane 'MNQP' with no connectivity between them. Mesh refinement is carried out for satisfactory convergence. Different crack densities (d/S) have been simulated by varying the half crack spacing (S). In the finite element model $u_z = 0$ for all nodes on the face ABCD and $u_y = 0$ for all nodes on the face HLKG. Load is applied in y direction via uniform applied displacements through the $\pm\theta$ layers ($\theta = 0^{\circ}, 15^{\circ}, 30^{\circ}, 45^{\circ}$ and 60°) over the face EIJF. Analysis is carried out for different crack lengths and for various crack densities.

Strain energies of the uncracked and cracked laminates are evaluated by summing up the strain energies of all the elements in the corresponding domain of analysis. The strain energy release rate is calculated using Eqn. (3) and variation of stiffness reduction in the width direction for different crack lengths has been calculated by using the Eqn. (9).

Results and Discussion

The transverse cracking behaviours in $[\pm\theta/90]_s$ angle-ply glass/epoxy laminates have been analysed for the following geometrical dimensions and applied displacements.

d = thickness of each 90° ply = 0.32 mm
b = total thickness of the two outer $\pm\theta$ plies = 0.32 mm
W = laminate width = 20 mm
Applied strain = 1%

Comparison of present 3D FE method with linear shearlag method [4] for stiffness reduction of $[\pm 45/90_2]_s$ CFRP laminates is shown in Fig. 3. Results estimated from the present 3D FE method are closer to the shearlag method for smaller crack densities. The marginal disagreement for higher crack densities may be due to the free edge effects of the laminates which have been neglected.

Strain Energy Release Rate

Fig. 4 shows the strain energy release rate G for $[\pm\theta/90^{\circ}]_s$ GL/E laminates as a function of crack density for different fibre orientation angles θ. It has been observed that the strain energy release rate increases with the increase in fibre angle θ of the outer plies and decreases with increasing crack densities.

Variations of strain energy release rates G in width direction for $[\pm 45^0/90^0]_s$ GL/E laminate with different crack densities are shown in Fig. 5. It is observed that the variation of strain energy release rate in width direction is negligible.

Loss in Stiffness

Fig. 6 shows the variation of stiffness reduction as the transverse crack grows towards full width of the laminate as a function of crack density for $[\pm\theta/90^0]_s$ GL/E laminate with different fibre orientation angles θ of the outer layers. It is seen that the stiffness reduction is increased with increase in fibre angle θ and crack density.

Variation of stiffness reduction in width direction for $[\pm\theta/90]_s$ GL/E laminate is shown in Fig. 7. In Fig. 7(a) the stiffness ratio $\left(\frac{E_c}{E_o}\right)$ is plotted as a function of crack length $\left(\frac{a}{W}\right)$ for four different crack densities and in Fig. 7(b) stiffness ratio $\left(\frac{E_c}{E_o}\right)$ is plotted against the crack densities for different crack lengths. Stiffness reduction has been noticed with increase in crack length and crack densities. However, stiffness reduction is small for lower crack densities as well as smaller crack lengths.

Conclusions

A 3d finite element method is used for calculating the stiffness loss and strain energy release rate for transverse cracked angle-ply laminates with varying crack densities and crack lengths. The effect of fibre angle θ on the transverse crack behaviour is also studied. The stiffness reduction increases as the crack extends in the width direction. Strain energy release rate decreases with increasing crack densities and fibre angle.

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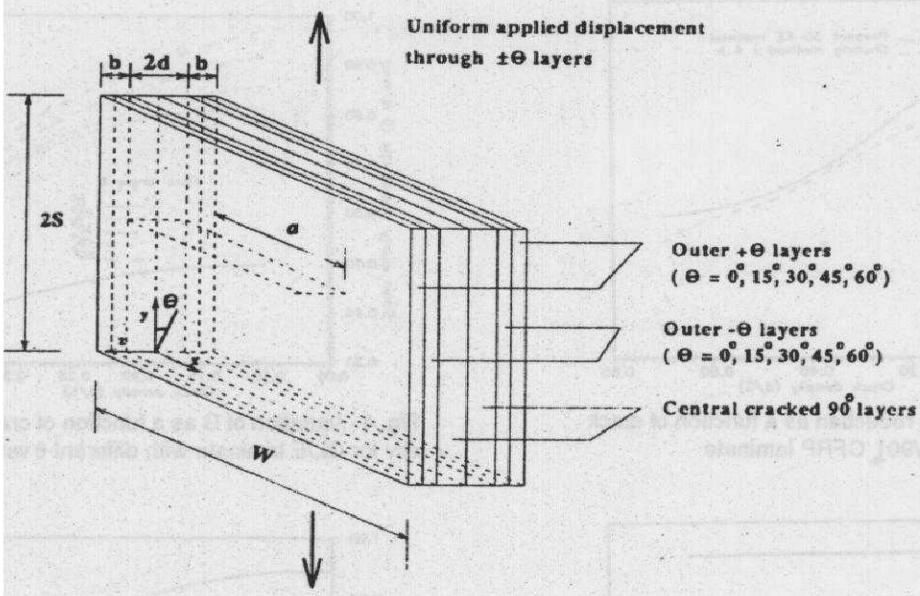

Fig. 1- Control volume of angle - ply composite laminate

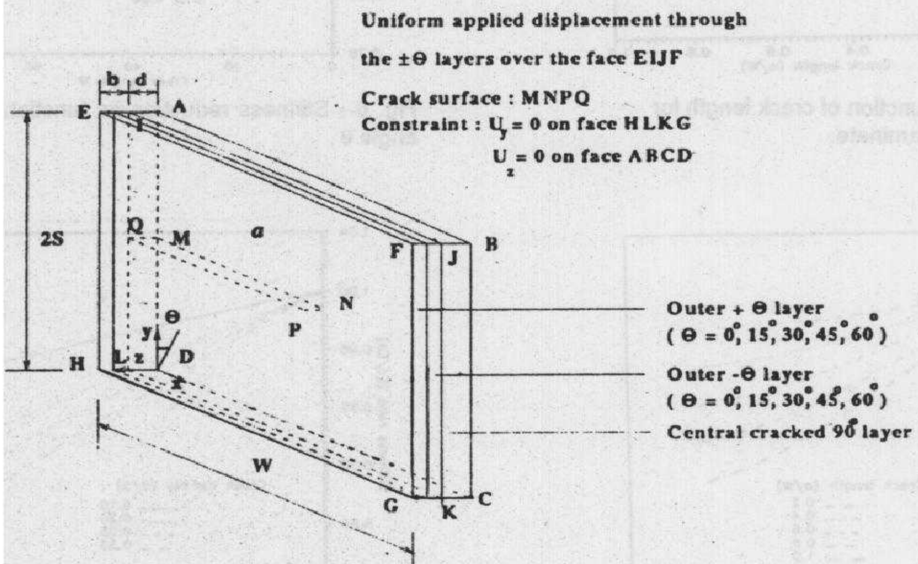

Fig. 2 - Symmetric half of the control volume

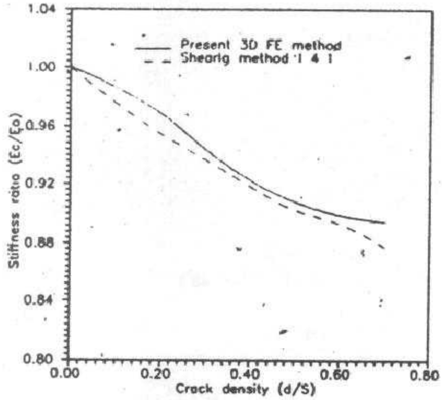

Fig. 3 - Stiffness reduction as a function of crack density for [±45/90]$_s$ CFRP laminate

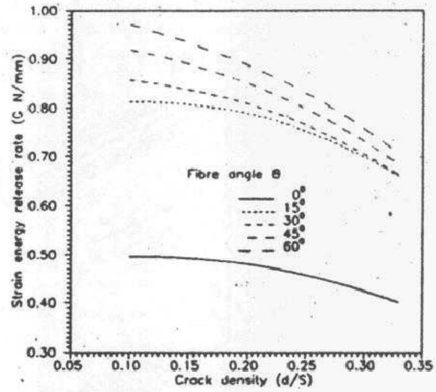

Fig. 4 - Variation of G as a function of crack density for GL/E laminate with different θ value.

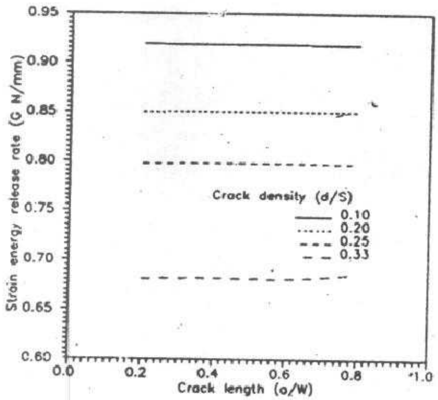

Fig. 5 - G as a function of crack length for [±45/90]$_s$ Gl/E laminate.

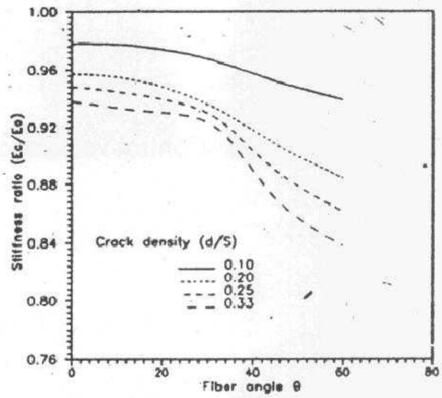

Fig. 6 - Stiffness reduction as function of fibre angle θ.

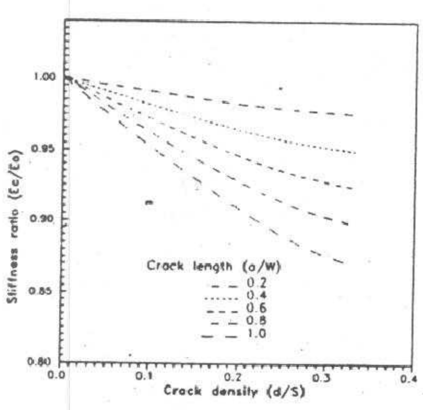

Fig. 7a - Variation of stiffness reduction in width for [±45/90]$_s$ GL/E laminate.

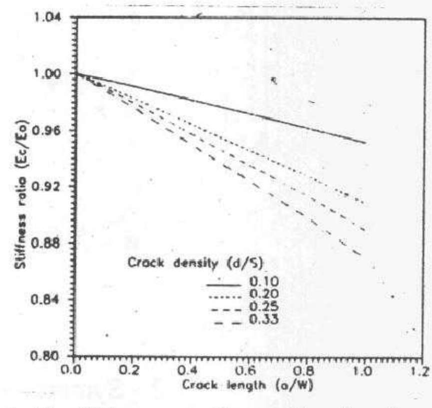

Fig. 7b - Stiffness reduction as a function of crack density for [±45/90]$_s$ GL/E laminate.