Mechanisms of fatigue failures

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ABSTRACT
Fatigue failures account for 90% of strength failures in engineering components. In order to analyse such failures and learn to prevent them in future, it is necessary to understand how and why fatigue failures occur. In this paper, an attempt has been made to contribute to such understanding. The concept of the weakest link in a structure initiating failure is discussed. It is shown that the weakest link principle can be extended to explain multiple origin of fatigue cracks. The micromechanisms that are responsible for the initiation of a fatigue crack and those that come into play during the growth process are highlighted. The formation of typical features of fatigue fractures, like striations and beachmarks, are discussed in relation to the mechanisms operative during fatigue crack growth. For the choice of materials for providing service under fatigue loading conditions, and to assess the integrity of components and structures already in service under such conditions, the fatigue resistance of material has to be quantified. The conventional approach to such quantification and the more recent fracture mechanics based differential approach to representing the fatigue resistance of materials are discussed.

INTRODUCTION
A component is said to have failed by fatigue when it disintegrates or collapses after having been subjected to a number of cycles of alternating stress. Usually no obvious damage or deterioration of its service capability can be observed throughout the majority of the loading cycles. The magnitude of the cyclic stress applied may be so small that their single application does not result in any detectable damage at all. And the failure surfaces are often apparently brittle, devoid, largely, of gross plastic deformation. Such observations led to the belief that components can become incapable of bearing load through “exhaustion” and prompted the usage of the term fatigue for such failures. With reference to the failure of components, fatigue can thus be defined as the phenomena leading to fracture under repeated or fluctuating stresses having a nominal maximum value.
less than the tensile strength, or even the yield strength, of the component material.

Although fatigue failures may seem to be abrupt, the process of fatigue fracture is progressive, beginning as minute cracks that grow during the service life of components. Submicroscopic changes take place in the crystalline structure of metals and alloys under the action of repetitive low-level load applications. These minute changes accumulate to lead to the formation of tiny microscopic cracks. These tiny cracks grow under cyclic loading into larger cracks. The large cracks continue to grow until the stress in the remaining ligament becomes unsustainable, whence fracture occurs during the application of the final load cycle.

In order to analyse and prevent fatigue failures, it is important to understand the micromechanisms that are operative during fatigue. It is also necessary to be able to identify the signatures of a fatigue fracture and to quantify the fatigue resistance of materials. These and other issues are discussed below.

THE WEAKEST LINK : THE SHOESTRING ANALOGUE

Stress, strength and failure are inseparably intertwined. Failure in a system will take place at a location where the stress exceeds the strength or resistance to failure. This location can be called the "weakest link" in the system. The situation may be illustrated by the case of failure of a shoelace. As shown in Fig.1, the expected location of breakage of a shoelace is at the upper eyelets. Failure at this location is expected because

Fig. 1 : Failure of a shoelace
i) the service stress is highest in this location as the lace is pulled tightest at the upper eyelets when the knot is tied, and

ii) the sliding motion at the upper eyelets is greatest during tightening; this leads to greater wear or abrasion at this location on the fibres of the lace.

If it is assumed that the shoelace has uniform mechanical properties along its length then it will eventually tear or break at the location where conditions are most severe, i.e. at the upper eyelets.

The approach adopted for a shoelace may be extended to complex components in service. All components will fail at the weakest link where the stress exceeds the intrinsic resistance to failure. However, it is to be remembered that the assumption of uniform mechanical properties may not be applicable for engineering components. At microscopic levels, there may be a great variation in the mechanical property of the material from which a component is made. Such variation may originate from defects in the microstructure such as inclusions, grain boundaries, etc. The variation of cross-sectional stress across a real component may likewise be substantial due to the complexity of geometries of such components.

Like all other mechanical failure, fatigue failures also occur at the weakest location. A further complicating factor is introduced in the case of fatigue by the gradually decreasing resistance of the material to fatigue failure with continuation of service. It still holds though that when the applied cyclic stress exceeds the residual fatigue strength, failure takes place.

The origination of fatigue failures may also be traced to a weak link. The situation may be exemplified by the case of a stepped shaft rotating under bending load. As shown in Fig.2(a), for a two-diameter shaft with a fillet radius, the maximum stresses would occur at the joining of the fillet with the smaller diameter due to the stress concentrating effect of the fillet radius. Hence it is logical to assume that the fatigue crack would originate at some point on the circle at this position. Each point on this circle is exposed to a sinusoidal variation of stress, as shown in Fig.2(b), as the shaft rotates under bending loads. The stress is zero when a point is at the neutral axis. As the point, in the course of rotation, reaches the top, it experiences maximum tensile stress. On continued rotation the stress decreases through zero to the maximum compressive stress when at the bottom position. The circle experiencing the severest cyclic stresses passes over millions of grains. Each of these grains would possess different strengths to resist fatigue failure (defined as the maximum stress amplitude at which the grains would not fail by fatigue for an infinitely large number of cycles). The variation in their fatigue strengths could arise from differences in orientation, differences in the
Fig. 2: Weakest link failure in a shaft rotating under bend load
defect structure within each grain and perhaps differences in local microstructure as well. The expected variation for a small number of consecutive grains are shown in Fig. 2(c). For a given stress amplitude $\Delta \sigma$ the fatigue resistance strengths of only a few of the grains may be overcome and the fatigue failure would originate from these “weak” grains preferentially. Various fatigue stress amplitude horizontals are superimposed on the fatigue resistance strengths shown in Fig. 2(b). For fatigue at amplitude $S_1$, none of the grains are being affected and fatigue failure may not take place. For fatigue at amplitude $S_2$, the fatigue resistance strength of only one grain is being overcome, and the fatigue crack would originate from this single point. If the fatigue amplitude is increased to $S_3$, then a number of grains will become sensitive and a fatigue failure originating from multiple points will occur.

THE MICROMECHANISM OF FATIGUE FAILURES

As discussed earlier, fatigue failures are progressive in nature. A scientific scrutiny of the process of fatigue failure reveals that it comprises of three distinct concatenated phases: (i) crack initiation, (ii) crack propagation and (iii) catastrophic failure on the attainment of a critical crack size.

The first two items in the list above constitute the **fatigue life** in any component, while the third is a final event, the avoidance of which is the prime objective of studying fatigue failures. The micromechanisms and characteristics of fatigue crack initiation and propagation are presented below.

Fatigue Crack Initiation

The process of fatigue crack initiation is highly complex and depends greatly on the plastic strain amplitude, the temperature, the deformation characteristics of the material (dislocation mobility, dislocation substructure etc.) and the material microstructure (degree of inhomogeneity). Klesnil and Lukás ([1]) give an extensive and instructive account of the various possibilities and the complexities involved.

For generalisation, the process may be subdivided into the following events:

1) generation or annihilation of the redundant dislocation density through fatigue hardening or softening to form a cyclically stabilized dislocation population

2) localisation of slip through the formation of constrained dislocation substructure, like persistent slip bands (PSB), dislocation cells or planar dislocation arrays, on further cycling
iii) interaction of this dislocation substructure with a free surface to produce extrusions and associated intrusions on it

iv) concentration of stresses by such intrusions to produce embryonic cracks.

It must be pointed out that the events outlined above have emerged from laboratory studies on homogeneous, single-phased materials mainly. In complex engineering materials, and in components, stress concentrators may be present from the outset, in the form of inclusions, casting or welding defects, grinding or machining marks for example, and then the process of fatigue crack initiation may be hastened due to faster localisation of slip. Such inhomogeneities, however, must also be conditioned by cyclic slip processes before they can initiate cracks. For example, cracks are found to initiate from grain boundaries only after incompatibility of slip exists across such boundaries and irreversible grain-boundary sliding can occur \(^{[2,3]}\). Similarly the stress concentration produced by an inclusion localises the slip processes which ultimately lead to the formation of a microcrack through decohesion of the inclusion-matrix interface or by cracking of the inclusion itself \(^{[4]}\). Sometimes crack-like defects may be present embedded in the material and the application of fatigue loading may lead to propagation of the crack straightaway without any need for initiation.

The model of crack initiation by localisation of cyclic slip culminates in the formation of an intrusion which, acting as a stress concentrator, nucleates an embryonic crack. However, the exact process by which an intrusion is transformed into a crack is a matter of debate. There is quite some inclination not to distinguish between an intrusion and a microcrack and to consider the cracking process as an extension of that which formed the intrusion itself \(^{[5-8]}\). Local brittle fracture ahead of an intrusion due to concentration of stress has been advanced as another possibility \(^{[9]}\). Condensation of vacancies generated during cyclic deformation on an intrusion has also been proposed \(^{[10]}\), although fatigue crack initiation at as low as -270°C \(^{[11]}\), a temperature where diffusion is practically impossible, restricts such vacancy models to high temperatures only. Amongst the other notable mechanisms that have been proposed are loss of coherency across slip planes due to defect accumulation \(^{[12]}\) and plastic instability occurring on a micro-scale at the root of an intrusion \(^{[13]}\).

**Fatigue Crack Propagation**

The initiation stage ends with the formation of a microcrack which usually lie along activated slip planes (intergranular cracks being exceptions), and are often numerous in any instance. Further growth of these cracks under cyclic loading occur through two distinct stages — Stage I and Stage II \(^{[14]}\), leading ultimately to
complete failure. Of the many microcracks that may exist simultaneously in a body, some do not propagate further at all, some propagate through Stage I before arresting or coalescing with other cracks, while usually a single microcrack grows to failure. The selection process involved in this phenomenon is governed by the activation available to each of the slip systems from the applied loading.

Stage I cracking is essentially extension of the microcracks along their habit planes, and therefore crystallographic in nature. Stage II propagation is typically non-crystallographic and occurs in a direction that is normal to the applied tensile stress axis. Because slip is a shear process, slip planes that are proximate to the planes of maximum shear in a component (±45° to the axis in an uniaxial test specimen) usually support Stage I growth. Hence, Stage I crack propagation is thought to be controlled by the shear component of the applied stress while Stage II cracking is controlled by the tensile component. However, experiments by Kaplan and Laird [15] in which artificial Stage I cracks failed to grow in copper single crystal cycled in compression (which would have the same shear component as a tensile loading), illustrated the importance of the tensile component of stress to the growth of Stage I cracks also.

Stage I cracks usually grow through a few grain diameters before deviating gradually into a Stage II crack. Various causes for this transition has been proposed — obstacles to easy glide blocking the crack tip, like an unfavourably oriented grain [14], conditions of constraint in the depth restricting slip due to a low shear stress to tensile stress ratio [16], the crack tip displacement accompanying the crack tip blunting during tensile straining exceeding the dislocation substructure size [17] etc. Generally lower stresses and a low mean stress, preferred orientations in the microstructure of a material and corrosive conditions promote Stage I growth. In exceptional circumstances Stage I growth may lead to final fracture [18].

The surface of a Stage I crack has a faceted appearance as the crack-path tilts on crossing grain boundaries. The surface of Stage II cracks is characteristically covered with parallel markings at intervals of the order of 0.1 µm or more called striations which are supposed to be successive positions of the crack front. The formation of striations is intimately connected with the micromechanism of fatigue crack propagation. Aspects of striations and micromechanisms proposed to be responsible for them are discussed later.

Crack propagation in Stages I and II is thought to be continuum controlled growth and is therefore relatively insensitive to the microstructure of the material. However, once the crack length increases sufficiently and incremental extensions are of the order of defect or particle spacing in a material, simple continuum behaviour is interrupted. Secondary cracking, void formation and other
static fracture modes become increasingly contributory and crack growth occurs by a combination of Stage II continuum controlled processes and these static modes. This situation is often designated as Stage III of fatigue crack propagation [17]. Such a state of growth becomes exceedingly important in characterizing fatigue lives of components made of high-strength materials and those operative at high temperatures.

Fig. 3 gives a schematic representation of the crack growth history. Starting from an intrusion/extrusion at the free surface, the crack grows in Stage I at a slant, probably in a crystallographic fashion. Gradually it deflects into a Stage II crack when a striation forming mechanism dominates. Further on in Stage II, static fracture modes are superimposed on the growth mechanism, till finally it fails catastrophically by shear at an angle to the growth direction.

**SIGNATURES OF FATIGUE FAILURES**

Fatigue failures can be identified by some typical features of/on the fracture surface. The characteristics of such features, the mechanisms responsible for their formation and the informations that can be gleaned from a study of such
features are detailed below. It must be pointed out that fatigue fracture surfaces must be carefully handled so that the signatures of fatigue are well preserved.

**Lack of Deformation**

An easy process of breaking a wire is by inducing a fatigue fracture in it by completely reverse bending it several times. The number of load reversals required are usually relatively few, say ten, and the stresses imposed during bending are very high. Such failures are extreme examples of high stress, high strain, low-cycle fatigue. If the fracture surface is observed under a microscope, it will be seen to be highly distorted, accompanied by large amounts of deformations along the periphery.

Engineering fatigue failures however are rarely of the type described above. As a matter of fact, they usually show very little macroscopic deformation. This is because most engineering fatigue failures occur at relatively low stress amplitudes, through a number of cycles. This type of failures are known as high-cycle fatigue failures.

The lack of deformations in fatigue failures is manifested by a macroscopically flat fracture surface. Microscopically, however, there is substantial amounts of plastic deformation that occur during the growth of a fatigue crack. Large amounts of deformation can also occur in the remaining ligament during final separation, if it is allowed to occur. Deformations of the fatigue fracture surface may take place due to post-fracture damage. Frequently projections from the mating fracture surfaces come into contact due to continued operation of a failed component and get damaged in the process.

**Fatigue Striations**

As noted earlier, striations are parallel markings at intervals of the order of 0.1µm or more, which delineate successive positions of a progressing fatigue crack front. Fatigue striations were first reported in the literature by Zapffe and Worden [19] although the term *striations* was not coined until later by Forsyth and Ryder [20]. Forsyth and Ryder also demonstrated that each striation was produced by a single load cycle through programmed amplitude loading. Since then it has been postulated that striation spacing is proportional to the amplitude of loading and the maximum applied load, there existing a sequencing effect for random amplitudes with variable maximum load [21,22]. However, a one-to-one correspondence between striation and load cycle does not necessarily exist. And recent investigations equipped with advanced methods of observation and measurement seem to uphold such a conclusion [23]. Evidence suggests that striation formation is restricted to crack growth rates approximately between $10^{-5}$ to $10^{-3}$ mm/cycle.
and a one cycle per striation mechanism is only observed at growth rates above \(10^{-5} - 10^{-4}\) mm/cycle \(^{[25]}\).

Striations are sometimes poorly defined and therefore often unrecognisable. This is particularly true in case of steels. Striation appearance may be affected by post-cracking deformation of the fracture surfaces \(^{[26]}\) and also by the environment. For example, they may be obliterated by oxidizing or corroding conditions \(^{[27,28]}\). In the absence of an environment, i.e. in vacuum, striations have been found not to form at all in metals \(^{[29]}\), although polymer fracture surfaces are completely covered with them \(^{[30]}\). It has been reported that striations are absent in tests conducted in inert atmospheres, like dry argon \(^{[31]}\). But other evidence would suggest that it is really a matter of degree and growth markings, though much less distinct than those observed in air, are found in dry argon at high magnifications \(^{[32]}\). A similar case is true for aluminium alloys tested in liquid nitrogen \(^{[33]}\).

Fatigue striations can be classified into two types — ductile and brittle. Early concepts of striations perceived them to consist of successive zones of ductile and brittle fractures. And striations exhibiting a pronounced brittle fracture zone were termed brittle \(^{[34]}\). Brittle striations are characterized by flat fracture facets with riverlines running parallel to the direction of crack propagation. They often lie on crystallographic planes. They are thought to contain contributions from cleavage failure modes as opposed to a plastic deformation mechanism giving rise to ductile striations. The formation of brittle striations is promoted by corrosive conditions \(^{[14,31,34-36]}\), low frequency \(^{[14,28,36,37]}\) and low values of stress intensity \(^{[37,38]}\). Mechanisms of formation of brittle striations tend to be modifications of those of ductile striations.

Fatigue striations provide a complete history of the successive positions of the fatigue crack front. Information on the shape of striations have been mainly obtained via metallographic sections through the crack tip \(^{[35,36,39-41]}\) and by electron-fractographic examination of failure surface replicas \(^{[21,22]}\). Fig.4 shows

![Striation morphologies reported in literature](image)
schematic representations of the various striation morphologies that have been propounded. Generally, a situation in which the undulations on the two surfaces of a crack are in anti-register seem to find more favour than one where the peaks and crevices are matching. The varied nature of striation morphologies indicate the variability of the mechanism of fatigue crack propagation. A number of mechanisms by which fatigue cracks grow have been advanced by researchers trying to unravel the mysteries of fatigue. A complete description of the models proposed is not within the scope of the present paper. In Fig.5, some of the major models have been schematically illustrated, alongwith the mechanisms by which formation of striations is supported. It must be remembered that there are a great many variants of each model which explain the observance of specific profiles and features.

Although striations are one of the characteristic features of fatigue failure surfaces, they are often not visible. Striations are usually not observable in very hard or very soft materials. In hardened steels, striations are either absent or poorly formed, probably because of their limited ductility. At the other extreme, soft metals are too vulnerable to obliteration of surface relief through damage by
contact etc. to show striations. Sometimes directional scouring of one fracture surface by the other produces parallel ridges which resemble striations, but are actually artifacts. In certain lamellar microstructures, such as pearlite, the fracture surface follows the weak planes of the underlying microstructure and thereby tends to exhibit striation-like features. It must be understood that due to the expected microscopically close spacing of striations, they are generally observable only at high magnifications, often with the help of the SEM.

Beachmarks

Beachmarks are the most characteristic feature of fatigue fracture surfaces and a positive means to identify them. The term beachmark is synonymous with stop marks, arrest marks, clamshell marks and conchoidal marks, all of which attempt to describe the appearance of fatigue surfaces.

Beachmarks are macroscopically visible ridges that represent the position of the fatigue crack front during interruptions in the propagation period. They are formed mainly due to enhanced corrosion or oxidation of the highly strained front of the fatigue crack which has temporarily been arrested due to stoppage of stress cycling or the imposition of overload cycles of stress. Sometimes when there is a drastic reduction in the applied stress cycle amplitude, the growth rate of a fatigue crack may get substantially retarded, due to essentially an overload effect, and a beachmark may be formed. Likewise a high overload in itself may result in local deviation of the crack path, producing a ridge at the location as the crack deviates back to the original growth direction. Similarly a sudden variation in the corrosiveness of the environment in which a fatigue crack is growing would result in a change in the nature of the corrosion product being formed at the crack tip and cause the appearance of a beachmark. If fatigue crack growth has been occurring continuously in an unchanging environment under constant or slightly varying load amplitudes, beachmarks will not form.

Beachmarks may often be confused with striations. In fact beachmarks and striations can be present on the same fracture surface, with many thousands of microscopic striations existing between each pair of macroscopic beachmarks. As discussed earlier, the process of formation of beachmarks and striations are quite different, although both represent the advance of the fatigue crack front. Fig.6 shows a schematic of the situation in a simplified form. Only a few striations, and not the thousands or millions that are present in a real situation, are shown between beachmarks. In the figure, ratchet marks, which are actually traces of vertical planes separating fatigue fractures originating from multiple initiation points, are also shown.
QUANTIFICATION OF FATIGUE RESISTANCE

In order to be able to compare materials with respect to their resistance to fatigue failures, or to confirm the acceptability of a given material for service under a known cyclic load, it is imperative to express fatigue resistance quantitatively. Conventionally fatigue resistance has been expressed in terms of the S-N curve and the Coffin-Manson relationship. More recently, with the evolution of damage tolerant design philosophy, wherein the presence of crack-like defect does not necessarily mean that a structural component is at, or even near, the end of its useful life, a fracture mechanics based quantification of fatigue crack propagation has become popular. The various approaches to quantification are discussed below.

Conventional Approach to Quantification of Fatigue Resistance

The conventional approach to fatigue acknowledges the applied stress amplitude, $\Delta \sigma$, as the crack driving force, while the time dependence of fatigue is characterized by the time or number of cycles to failure. The concept of the S-N curve and the Coffin-Manson relationship are examples of this category.

The S-N Curve

The first systematic research on fatigue was carried out by Wöhler [51]
between the years 1852-1871. He investigated the failure of railway wheel axles under stress controlled cyclic loading. Wöhler showed that the fatigue life, i.e., number of cycles to failure \( N \) or \( N_f \), was primarily dependent on the applied stress range, \( 'S' \). The fatigue life was seen to increase with a decrease in the applied stress range, resulting in the typical S-N curve shown in Fig. 7.

As shown in Fig. 7, there are two types of S-N curves. In one case (curve A), usually derived for strain-ageing materials such as steels, a sharp knee is demonstrated at a particular value of stress range known as the fatigue limit. This related to the dynamic strain-ageing behaviour of such materials. The fatigue life is effectively infinite below this fatigue limit stress range. In the other case (curve B), produced by non-strain-ageing materials like non-ferrous alloys, such sharp fatigue limit is apparent. However, for design purposes, an endurance limit is defined for such materials as the value of the stress range at which the fatigue life is \( 10^8 \) cycles. A general definition of the fatigue limit is that it represents the highest stress level at which the competitive processes of dislocation multiplication, cyclic hardening and strain ageing are at equilibrium. If N type data from above the fatigue limit is plotted using logarithmic axes, then it is seen that they fall on a straight line which could be represented by the equation:

\[
\log \Delta \sigma = -\alpha_1 \log N_f + \log C_1
\]

or

\[
N_f^{\alpha_1} \Delta \sigma = C_1
\]
where $\alpha_i$ and $C_i$ are constants. Such an equation can then be used to predict the failure of components under a specified stress amplitude, provided the specimens used to generate the S-N curve are exact replicas of the components and experience conditions comparable to them.

Mean stress can represent an important test variable in the evaluation of a material's fatigue response. Hence various empirical relations have been developed to calculate the equivalent stress range at a non-zero mean stress from that at a zero mean stress, for a given fatigue life. The Goodman, Gerber and Soderberg relations\(^{53}\) are examples of such empirical equations.

S-N curves still enjoy considerable engineering use. Concepts of similitude, extrapolation and scaling are important in using them for design purposes.

**The Coffin-Manson Relationship**

The failure of a component, as characterized by the S-N curve, is essentially under load controlled cycling. However, in practical situations, a structure may be subjected to strain or position controlled cycling, for example due to temperature fluctuations, and undergo failure under such circumstances. Coffin\(^{54}\) and Manson\(^{55}\) were pioneers in presenting experimental data from constant plastic strain range ($\Delta \varepsilon_p$) tests. They proposed what is now known as the Coffin-Manson relationship to deal with such strain controlled situations which can be written as

$$N^\alpha \Delta \varepsilon_p = C^2 \ldots \ 2$$

in similarity to Eq.1, where $\alpha_2$ and $C_2$ are constants.

In fact, it is believed that it is the plastic strain existing locally in a specimen which determines its fatigue behaviour. When testing under stress controlled cycling, the applied strain range decreases or increases (depending upon whether the material cyclically hardens or softens) and hence it can be argued that data from strain controlled tests are more suitable for correlating fatigue behaviour and fatigue properties.

Eq. 2 has been found to provide a good fit for data from a wide range of alloys\(^{56}\). A value of $\alpha_2 \approx 0.5$ has been proposed to be valid for many materials\(^{57}\), while $C_2$, termed as the *fatigue ductility coefficient* has been often found to be of the same order as the true fracture strain, $\varepsilon_f$, obtained from tensile tests\(^{11}\).

A plastic strain fatigue limit may be obtained from a Coffin-Manson type plot, just as in the case of the S-N curve. It has been found in single crystal
studies [56, 58-61] that there exists a well-defined lower limit of plastic strain below which there is insufficient irreversibility in dislocation movement for the formation of PSBs and subsequent crack initiation. The extension of this concept to polycrystalline materials can be viewed as being equivalent to the plastic strain fatigue limit [1, 56]. It therefore represents the transition between reversible and irreversible dislocation processes at saturation. The plastic strain fatigue limit has been demonstrated to be equivalent to the fatigue limit determined in stress-controlled tests by using the cyclic stress-strain curve [62].

**Inadequacies of the Conventional Approach**

Engineering design is still largely based on failure data produced by the conventional approach. Conventional smooth specimen testing still provides important informations on crack initiation and the initial stage of propagation. However, in spite of its usefulness, the conventional approach is inadequate in many respects which are enumerated below.

Firstly, it does not distinguish between the initiation and propagation stage of crack growth. Propagation controlled behaviour, which account for the majority of failures, cannot, therefore, be modelled by this approach. Secondly, the conventional approach inherently recognizes only two points in the history of the formation and growth of a crack in a body — the initial condition of no crack and the final condition of a totally cracked (and failed) body. It does not provide any information about the velocity and acceleration of the crack as it grows. This is unacceptable from the point of view of a modern design code which includes monitoring of the progression of cracks for the maintenance of safety.

And finally, the conventional approach does not provide any link between the micromechanisms of crack propagation that have been advanced and the fatigue of materials. Hence the basic aspiration of explaining fatigue from fundamental considerations is not fulfilled in the least.

**Differential Approach to Quantification of Fatigue Resistance**

In the differential approach to quantification, the rate of the process of fatigue crack growth, $da/dN$ ($N=$ no. of cycles), is expressed as a function of a crack driving force. The early differential laws of crack growth used a product of the stress amplitude, $\Delta\sigma$, and the instantaneous crack length, $a$, to represent the crack driving force. In 1963, Paris and Erdogan [63] made a critical analysis of the growth laws then available and proposed the use of Irwin’s [64] stress intensity factor (SIF), $K$, for characterizing crack growth rate. The SIF is the most commonly used fracture mechanics parameter, and presently the differential approach towards quantification of fatigue resistance involves the usage of this parameter.
fracture mechanics parameter.

Paris and Erdogan showed that while the various differential fatigue crack growth laws could be validated by a limited set of experimental data by employing different plotting techniques, they were unable to correlate a larger amount of data from various sources. The use of the stress intensity factor range, $\Delta K$ (in similarity to $\Delta \sigma$), instead of a combination of $\Delta \sigma$ and $a$, seemed to provide a better fit, and Paris and Erdogan proposed the relationship

$$\frac{da}{dN} = C \Delta K^4$$

... 3
to characterise fatigue crack growth. As per their contention, $C$ seemed to be independent to the material and therefore enhanced the appeal of their relationship. However, this has since been found not to be strictly true; nor is the exponent of $\Delta K$ always equal to 4. The Paris-Erdogan relationship has been widely substantiated by a number of experimental investigations \cite{165-181}. However, current status of knowledge indicate that the exponent can assume a number of values \cite{169}, and a general form of the Paris-Erdogan relationship can be stated as

$$\frac{da}{dN} = C \Delta K^m$$

... 4

where $m$ varies with the situation. In the above equation, $\Delta K$ for any component can be related to $\Delta \sigma$ through a relation of the form

$$\Delta K = \Delta \sigma \sqrt{\pi a} Y$$

... 5

where $Y$ is a function of the geometry and the crack length contained in it. Further, Eq.4 is universally applicable to specimens and components, and therefore fatigue crack growth data obtained from specimens can be used to predict or compare the situation in components through judicious use of the function $Y$.

If the rate of fatigue crack growth per cycle, $da/dN$, is plotted in logarithmic scales against the stress intensity factor (SIF) range, $\Delta K$, attending the fatigue loading cycle, then for the entire range of fatigue crack growth rates, from $10^{-8}$ to $10^{-3}$ mm/cycle in a typical instance, a sigmoidal curve, shown schematically in Fig.8, is obtained. Such a plot may be sub-divided into three regimes of crack growth as shown in the figure. The Paris-Erdogan relationship, discussed in the earlier section, applies to regime $B$ only. In regime $C$, the linearity of the curve breaks down as the maximum load in the fatigue cycle approaches the load sustainable for a given fracture toughness of the material for the instantaneous crack
Fig. 8: Typical fatigue crack growth rate curve showing the three regimes of crack growth length. Similarly, in regime $A$, a non-linearity is observed as the $\Delta K$ approaches minimum value. This minimum value is called the threshold $\Delta K$, designate $\Delta K_{th}$, and represents the crack driving force below which crack growth is virtually undetectable. Regime $A$ is therefore known as the threshold regime. In general, crack growth rates in regime $B$ are relatively insensitive to limited variations in microstructure, frequency, environment and mean stress. The same factors, however, have large influences in the threshold regime.

In a simplistic sense it can be said that $\Delta K_{th}$, is to crack growth what fatigue strength or endurance limit, $\Delta \sigma_o$, is to fatigue life for uncracked material. However, it must be understood that $\Delta K_{th}$ is not usually definable by or obtainable from $\Delta \sigma_o$ because $\Delta \sigma_o$ is a limit on crack initiation while $\Delta K_{th}$ is a limit on crack growth.

As mentioned earlier, threshold regime crack growth has been found to be sensitive to microstructure. This is because at the low rates of crack growth obtained in the regime, the microstructure can be of a scale comparable with the crack tip micro-mechanics. A recent review by Bulloch [70] highlights the influence of microstructure on threshold regime crack growth. The importance of the relationship between microstructure and $\Delta K_{th}$ can be understood from Fig. 4 taken from Lindley and Nix [71]. It can be seen that in the process of strengthening a material through grain refining, precipitation hardening etc., although the fatigue strength (based on S-N data) improves, the $\Delta K_{th}$ decreases. Hence, although the resistance to crack initiation increases through microstructural strengthenin processes, the resistance to crack propagation from pre-existing defects (whic
Fig. 9: Dependence of threshold SIF, $\Delta K_{th}$, and fatigue strength, $\Delta \sigma_f$, on material strength are present in all materials) actually decreases. Due to crack growth being microstructurally sensitive in the threshold regime, the crack morphology is often crystallographic in nature.

Just as the Paris-Erdogan relation is used to represent regime B crack growth, for the threshold regime various empirical laws are available to describe the growth of fatigue cracks. One such relation is that due to Beevers and Carlson\textsuperscript{1721}, stated as

$$\frac{da}{dN} = C (\Delta K - \Delta K_{th})^n$$ \hfill (6)

The differential crack propagation laws can usually be integrated to produce a form equivalent to the S-N curve equation or the Coffin-Manson relation. Thus they are able to model endurance behaviour as depicted by the conventional approach. At the same time, these laws can predict crack propagation at stress levels below the fatigue limit originating from artificial or pre-existing cracks — a case untenable with the conventional approach. Hence the equivalence of growth mechanism for high stress and low stress, high-cycle and low-cycle situations can be demonstrated, thereby negating the need for such artificial divisions.

**CONCLUDING REMARKS**

In this paper, a brief discourse on the micromechanisms and characteristics of fatigue failures and the quantification of fatigue resistance of materials has been presented. The emphasis has been on understanding why, how and where fatigue
failures occur. Another approach to understanding fatigue failures, with a view to gaining expertise in analysing such failures, is to get exposed to typical case studies of failure under fatigue loading and get acquainted with fatigue fracture surfaces. For such an approach, the reader is directed to excellent case studies and fractographs presented in Vol.12 of the 9th edition \cite{73} and Vol.10 of the 8th edition \cite{74} of the Metals Handbook, and the ASM source books and technical reports on failure analysis \cite{75-77}. In closure it may be said that although reportedly 90% of engineering failures occur due to fatigue, scientific scrutiny of fatigue failures indicate that the majority of such failures could have been averted with better application of the knowledge at hand.

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